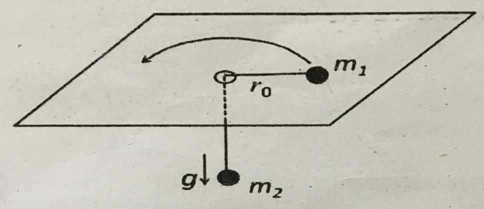
CSIR-NET June 2019 (Solutions to some selected questions)

Q1:

1. Two particles of masses m_1 and m_2 are connected by a massless thread of length ℓ as shown in figure below.



The particle of mass m_1 on the plane undergoes a circular motion with radius r_0 and angular momentum L. When a small radial displacement ϵ (where $\epsilon \ll r_0$) is applied, its radial coordinate is found to oscillate about r_0. The frequency of the oscillations is

1. $\sqrt{\frac{7m_2 g}{(m_1+\frac{m_2}{2})r_0}}$
2. $\sqrt{\frac{7m_2 g}{(m_1+m_2)r_0}}$
3. $\sqrt{\frac{3m_2 g}{(m_1+\frac{m_2}{2})r_0}}$
4. $\sqrt{\frac{3m_2 g}{(m_1+m_2)r_0}}$

Ans:

Two mass points m_1 and m_2 ($m_1 \neq m_2$) are connected by a string of length l passing through a hole in a horizontal table. The string and mass points move without friction with m_1 on the table and m_2 free to move in a vertical line.

(a) What initial velocity must m_1 be given so that m_2 will remain motionless a distance d below the surface of the table?

(b) If m_2 is slightly displaced in a vertical direction, small oscillations ensue. Use Lagrange's equations to find the period of these oscillations.

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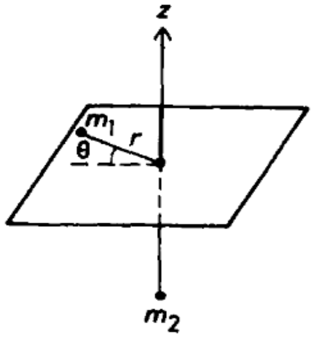

(a) m_1 must have a velocity v perpendicular to the string such that the centripetal force on it is equal to the gravitational force on m_2:

$$\frac{m_1 v^2}{l-d} = m_2 g \ ,$$

or

$$v = \sqrt{\frac{m_2(l-d)g}{m_1}} \ .$$

(b) Use a frame of polar coordinates fixed in the horizontal table as shown in Fig. 2.19. m_2 has z-coordinate $-(l-r)$ and thus velocity $\dot{r}$. The Lagrangian of the system is then

$$L = T - V = \frac{1}{2}m_1(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}m_2\dot{r}^2 + m_2 g(l-r) \ .$$

Lagrange's equations give

$$m_1 r^2 \dot{\theta} = \text{constant} \ ,$$
$$(m_1 + m_2)\ddot{r} - m_1 r\dot{\theta}^2 + m_2 g = 0 \ .$$

and the above equation becomes

$$\ddot{\rho} + \frac{3m_2 g}{(m_1+m_2)(l-d)}\rho = 0 \ .$$

Hence ρ oscillates about O, i.e. r oscillates about the value $l-d$, with angular frequency

$$\omega = \sqrt{\frac{3m_2 g}{(m_1+m_2)(l-d)}} \ ,$$

or period

$$T = 2\pi\sqrt{\frac{(m_1+m_2)(l-d)}{3m_2 g}} \ .$$

Source Y. K. Lim

Q2:

A charged, spin-less particle of mass m is subjected to an attractive potential $V(x,y,z) = \frac{1}{2}k(x^2 + y^2 + z^2)$, where k is a positive constant. Now a perturbation in the form of a weak magnetic field $\boldsymbol{B} = B_0\hat{k}$ (where B_0 is a constant) is switched on. Into how many distinct levels will the second excited state of the unperturbed Hamiltonian split?

1. 5
2. 4
3. 2
4. 1

Answer:

(Reference Question)

A particle of mass M, charge e, and spin zero moves in an attractive potential $k(x^2 + y^2 + z^2)$. Neglect relativistic effects.

(a) Find the three lowest energy levels E_0, E_1, E_2; in each case state the degeneracy.

(b) Suppose the particle is perturbed by a small constant magnetic field of magnitude B in the z direction. Considering only states with unperturbed energy E_2, find the perturbations to the energy.

Reference Answer (Y. K. Lim)

(b) For a weak magnetic field B in the z direction, the perturbation Hamiltonian is

$$H' = -\frac{eB}{2Mc}\hat{L}_z .$$

Then in spherical coordinates we have

$$E_{nlm} = E_{nl} - \frac{eB}{2Mc}m\hbar ,$$

where $m\hbar$ is the eigenvalue of $\hat{L}_z$. Thus the different degenerate states of E_2 have perturbed energies:

$$E_{200} = E_{20}$$

$$E_{222} = E_{22} - \frac{eB}{Mc}\hbar,$$

$$E_{221} = E_{22} - \frac{eB}{2Mc}\hbar,$$

$$E_{220} = E_{22},$$

$$E_{22-1} = E_{22} + \frac{eB}{2Mc}\hbar,$$

$$E_{22-2} = E_{22} + \frac{eB}{MC}\hbar.$$

It is seen that the degeneracy is partially destroyed.