JEST 2014

PART A: THREE MARK QUESTIONS

1. A dynamical system with two generalized coordinates q_1 and q_2 has Lagrangian $L = \dot{q}_1^2 + \dot{q}_2^2$. If p_1 and p_2 are the corresponding generalized momenta, the Hamiltonian is given by

   (a). $(p_1^2 + p_2^2)/4$

   (b). $(\dot{q}_1^2 + \dot{q}_2^2)/4$

   (c). $(p_1^2 + p_2^2)/2$

   (d). $(p_1\dot{q}_1 + p_2\dot{q}_2)/4$

2. In a certain inertial frame two light pulses are emitted, a distance 5 km apart and separated by 5 ‾s. An observer who is traveling, parallel to the line joining the points where the pulses are emitted, at a velocity V with respect to this frame notes that the pulses are simultaneous. Therefore V is

   (a). $0.7c$

   (b). $0.8c$

   (c). $0.3c$

   (d). $0.9c$

3. Suppose a spin 1/2 particle is in the state

$$|\psi\rangle = \frac{1}{\sqrt{6}}\begin{bmatrix} 1+i \\ 2 \end{bmatrix}$$

   If S_x (x component of the spin angular momentum operator) is measured what is the probability of getting $+\hbar/2$?

   (a). $1/3$

   (b). $2/3$

   (c). $5/6$

   (d). $1/6$

4. For an optical fiber with core and cladding index of $n_1 = 1.45$ and $n_2 = 1.44$, respectively, what is the approximate cut-off angle of incidence? Cut-off angle of incidence is defined as the incidence angle below which light will be guided.

   (a). $7°$

   (b). $22°$

   (c). $5°$

   (d). $0°$

5. A double pendulum consists of two equal masses m suspended by two strings of length l. What is the Lagrangian of this system for oscillations in a plane? Assume the angles θ_1, θ_2 made by the two strings are small (you can use $\cos\theta = 1 - \theta^2/2$).

   Note: $\omega_0 = \sqrt{g/l}$.

   (a). $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

   (b). $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

   (c). $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

   (d). $L \approx ml^2\left(\frac{1}{2}\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \omega_0^2\theta_2^2\right)$

6. Circular discs of radius 1 m each are placed on a plane so as to form a closely packed triangular lattice. The number of discs per unit area is approximately equal to

   (a). 0.86 m^{-2}

   (b). 0.43 m^{-2}

   (c). 0.29 m^{-2}

   (d). 0.14 m^{-2}

7. What are the solutions to $f''(x) - 2f'(x) + f(x) = 0$?

   (a). c_1e^x/x

   (b). $c_1x + c_2/x$

   (c). $c_1xe^x + c_2$

   (d). $c_1e^x + c_2xe^x$

8. An ideal gas of non-relativistic fermions in 3-dimensions is at 0 K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor

   (a). $2^{1/2}$
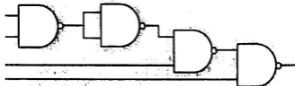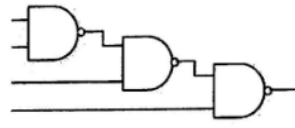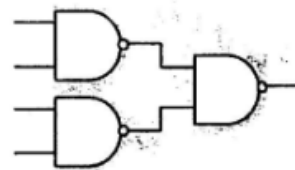
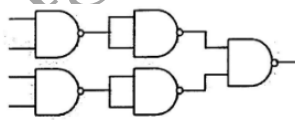   (b). 1

   (c). $2^{1/3}$

   (d). $2^{-1/3}$

9. The value of $\int_{0.2}^{2.2} xe^x dx$ by using the one-segment trapezoidal rule is close to

   (a). 11.672
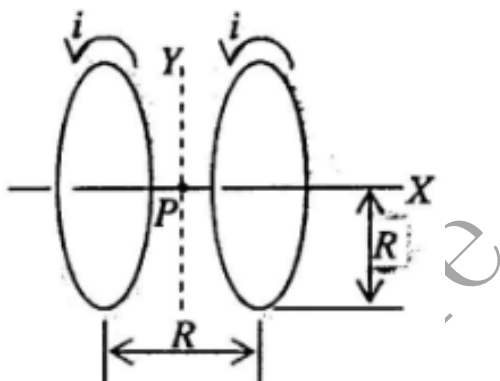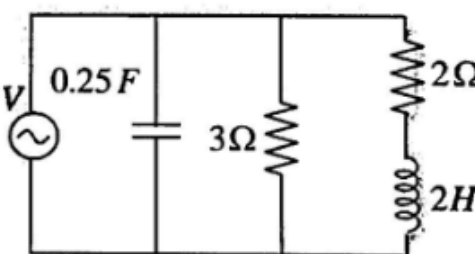
   (b). 11.807

   (c). 20.099

   (d). 24.119

10. The Hamiltonian operator for a two-state system is given by

$$H = \alpha(|1\rangle\langle1| - |2\rangle\langle2| + |1\rangle\langle2| + |2\rangle\langle1|),$$

    where α is a positive number with the dimension of energy. The energy eigenstates corresponding to the larger and smaller eigenvalues respectively are:

(a). $|1\rangle - (\sqrt{2}+1)|2\rangle, |1\rangle + (\sqrt{2}-1)|2\rangle$

(b). $|1\rangle + (\sqrt{2}-1)|2\rangle, |1\rangle - (\sqrt{2}+1)|2\rangle$

(c). $|1\rangle + (\sqrt{2}-1)|2\rangle, (\sqrt{2}+1)|1\rangle - |2\rangle$

(d). $|1\rangle - (\sqrt{2}+1)|2\rangle, (\sqrt{2}-1)|1\rangle + |2\rangle$

11. Given the fundamental constants $\hbar$ (Planck's constant), G (universal gravitation constant) and c (speed of light), which of the following has dimension of length?

    (a). $\sqrt{\frac{\hbar G}{c^3}}$

    (b). $\sqrt{\frac{\hbar G}{c^5}}$

    (c). $\frac{\hbar G}{c^3}$

    (d). $\sqrt{\frac{\hbar G}{8\pi G}}$

12. Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|l, m\rangle$. Let $A = \hat{n} \cdot \vec{L}$ denote an operator, where $\hat{n}$ is a unit vector parametrized in terms of two angles as $(n_x, n_y, n_z) = (\sin\theta\cos\phi, \sin\phi, \cos\theta)$. The width ΔA in $|l, m\rangle$ state is:

    (a). $\sqrt{\frac{l(l+1)-m^2}{2}}\hbar\cos\theta$

    (b). $\sqrt{\frac{l(l+1)-m^2}{2}}\hbar\sin\theta$

    (c). $\sqrt{l(l+1)-m^2}\hbar\sin\theta$

    (d). $\sqrt{l(l+1)-m^2}\hbar\cos\theta$

13. The Laplace transformation of $e^{-2t}\sin 4t$ is

    (a). $\frac{4}{s^2+4s+25}$

    (b). $\frac{4s}{s^2+4s+20}$

    (c). $\frac{4}{s^2+4s+20}$

    (d). $\frac{4s}{2s^2+4s+20}$

14. In the mixture of isotopes normally found on the earth at the present time, $^{238}_{92}$U has an abundance of 99.3% and $^{235}_{92}$U has an abundance of 0.7%. The measured lifetimes of these isotopes are 6.52×10^9 years and 1.02×10^9 years, respectively. Assuming that they were equally abundant when the earth was formed, the estimated age of the earth, in years is

    (a). 6.0×10^9

    (b). 1.0×10^9

    (c). 6.0×10^8

    (d). 1.0×10^8

15. Which of the following circuits will act like a 4-input NAND gate?

    (a.)

    (b.)

    (c.)

    (d.)

16. Consider a three-state. system with energies E, E and $E - 3g$ (where g is a constant) and respective eigenstates $|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, |\psi_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. If the system is initially (at $t = 0$), in state $|\psi_i\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, what is the probability that at a later time t system will be in state $|\psi_f\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

    (a). 0

    (b). $\frac{4}{9}\sin^2\left(\frac{3gt}{2\hbar}\right)$

    (c). $\frac{4}{9}\cos^2\left(\frac{3gt}{2\hbar}\right)$

    (d). $\frac{4}{9}\sin^2\left(\frac{E-3gt}{2\hbar}\right)$

17. A hydrogen atom in its ground state is collided with an electron of kinetic energy 13.377 eV. The maximum factor by which the radius of the atom would increase is

(a). 7

(b). 8

(c). 49

(d). 64

18. The formula for normal strain in a longitudinal bar is given by $\varepsilon = \frac{F}{AE}$, where F is normal force applied, A is cross-sectional area of the bar and E is Young's modulus. If $F = 50 \pm 0.5$ N, $A = 0.2 \pm 0.002$ m^2 and $E = 210 \times 10^9 \pm 1 \times 10^9$ Pa, the maximum error in the measurement of strain is

(a). 1.0×10^{-12}

(b). 2.95×10^{-11}

(c). 1.2×10^{-9}

(d). 1.19×10^{-9}

19. A monoatomic gas consists of atoms with two internal energy levels, ground state $E_0 = 0$ and an excited state $E_1 = E$. The specific heat of the gas is given by

(a). $\frac{3}{2}k$

(b). $\frac{E^2 e^{E/kT}}{kT^2(1+e^{E/kT})^2}$

(c). $\frac{3}{2}k + \frac{E^2 e^{E/kT}}{kT^2(1+e^{E/kT})^2}$

(d). $\frac{3}{2}k - \frac{E^2 e^{E/kT}}{kT^2(1+e^{E/kT})^2}$

20. Two large nonconducting sheets, one with a fixed uniform positive charge and the other with a fixed uniform negative charge, are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities is $\sigma_+ = 6.8\mu C\,m^{-2}$ for the positively charged sheet and $\sigma_- = 4.3\mu C\,m^{-2}$ for the negatively charged sheet. What is the electric field in the region between the sheets?

(a). 6.30×10^5 N/C

(b). 3.84×10^5 N/C

(c). 1.40×10^5 N/C

(d). 1.16×10^5 N/C

21. A monochromatic wave propagates in a direction making an angle $60°$ with the x-axis in the reference frame of source. The source moves at speed $\nu = \frac{4c}{5}$ towards the observer. The direction of the (cosine of angle) wave as seen by the observer is

(a). $\cos\theta' = \frac{13}{14}$

(b). $\cos\theta' = \frac{3}{14}$

(c). $\cos\theta' = \frac{13}{6}$

(d). $\cos\theta' = \frac{1}{2}$

22. A system of two circular co-axial coils carrying equal currents I along same direction having equal radius R and separated by a distance R (as shown in the figure below). The magnitude of magnetic field at the midpoint P is given by

(a). $\frac{\mu_0 I}{2\sqrt{2}R}$

(b). $\frac{4\mu_0 I}{5\sqrt{5}R}$

(c). $\frac{8\mu_0 I}{5\sqrt{5}R}$

(d). 0

23. Find the resonance frequency (rad/sec) of the circuit shown in the figure below

(a). 1.41

(b). 1.0

(c). 2.0

(d). 1.73

24. The temperature of a thin bulb filament (assuming that the resistance of the filament is nearly constant) of radius r and length L is proportional to

(a). $r^{1/4}L^{-1/2}$

(b). $L^2 r$

(c). $L^{1/4}r^{-1}$

(d). $r^2 L^{-1}$

25. Ice of density ρ_1 melts at pressure P and absolute temperature T to form water of density ρ_2. The latent heat of melting of 1 gram of ice is L. What is the change in the internal energy ΔU resulting from the melting of 1 gram of ice?

(a). $L + P\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$ (b). $L - P\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$

(c). $L - P\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$ (d). $L + P\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$

PART B: ONE MARK QUESTIONS

26. A spherical air bubble is embedded in a glass slab. It will behave like a

    (a). Cylindrical lens
    (b). Achromatic lens
    (c). Converging lens
    (d). Diverging lens

27. The acceleration experienced by the bob of a simple pendulum is

    (a). maximum at the extreme positi6ns
    (b). maximum at the lowest (central) positions
    (c). maximum at a point between the above two positions
    (d). same at all positions

28. A 100 ohms resistor carrying current of 1 Amp is maintained at a constant temperature of $30°C$ by a heat bath. What is the rate of entropy increase of the resistor?

    (a). 3.3 Joules/K/sec
    (b). 6.6 Joules/K/sec
    (c). 0.33 Joules/K/sec
    (d). None of the above

29. Consider a Hamiltonian system with a potential energy function given by $V(x) = x^2 - x^4$. Which of the following is correct?

    (a). The system has one stable point
    (b). The system has two stable points
    (c). The system has three stable points
    (d). The system has four stable points

30. The lowest quantum mechanical energy of a particle confined in a one-dimensional box of size L is 2 eV. The energy of the quantum mechanical ground state for a system of three non-interacting spin $\frac{1}{2}$ particles is

    (a). 6 eV
    (b). 10 eV
    (c). 12 eV

(d). 16 eV

31. The value of elastic constant for copper is about $100~\text{Nm}^{-1}$ and the atomic spacing is 0.256 nm. What is the amplitude of the vibration of the Cu atoms at 300 K as a percentage of the equilibrium separation?

    (a). 4.55%
    (b). 3.55%
    (c). 2.55%
    (d). 1.55%

32. What is the contribution of the conduction electrons in the molar entropy of a metal with electronic coefficient of specific heat?

    (a). γT
    (b). γT^2
    (c). γT^3
    (d). γT^4

33. Consider a system of $2N$ non-interacting spin $1/2$ particles each fixed in position and carrying a magnetic moment μ. The system is immersed in a uniform magnetic field B. The number of spin up particle for which the entropy of the system will be maximum is

    (a). 0
    (b). N
    (c). $2N$
    (d). $N/2$

34. When two different solids are brought in contact with each other, which one of the following is true?

    (a). Their Fermi energies become equal
    (b). Their band gaps become equal
    (c). Their chemical potentials become equal
    (d). Their work functions become equal

35. For which gas the ratio of specific heats (C_p/C_v) will be the largest?

    (a). mono-atomic
    (b). di-atomic
    (c). tri-atomic
    (d). hexa-atomic

36. A ball bounces off earth. You are asked to solve this quantum mechanically assuming the earth is an infinitely hard sphere. Consider surface of earth as the origin implying $V(0) = \propto$ and a linear potential elsewhere (i.e. $V(x) = -mgx$ for $x > 0$). Which of the following wave functions is physically admissible for this problem (with $k > 0$):

(a). $\psi = e^{-kx}/x$

(b). $\psi = xe^{-kx^2}$

(c). $\psi = -Axe^{kx}$

(d). $\psi = Ae^{-kx^2}$

37. Which functional form of potential best describes the interaction between a neutral atom and an ion at large distances (i.e. much larger than their diameters)

    (a). $V \propto -1/r^2$

    (b). $V \propto -1/r$

    (c). $V \propto -e^{-r/a}/r$

    (d). $V \propto -1/r^3$

38. An electron is executing simple harmonic motion along the y-axis in right handed coordinate system. Which of the following statements is true for emitted radiation?

    (a). The radiation will be most intense in xz plane

    (b). The radiation will be most intense in xy plane

    (c). The radiation will violate causality

    (d). The electron's rest mass energy will reduce due to radiation loss

39. Two point objects A and B have masses 1000 Kg and 3000 Kg respectively. They are initially at rest with a separation equal to 1 m. Their mutual gravitational attraction then draws them together. How far from A's original position will they collide?

    (a). 1/3 m

    (b). 1/2 m

    (c). 2/3 m

    (d). 3/4 m

40. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been

    (a). Less

    (b). More

    (c). The same

    (d). Depends on the electron mass

41. Let us write down the Lagrangian of a system as $L(x, \ddot{x}, \dddot{x}) = mx\ddot{x} + kx^2 + cx\,\dddot{x}$. What is the dimension of c?

    (a). MLT^{-3}

    (b). MT^{-2}

    (c). MT

(d). ML^2T^{-1}

42. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$ for a well behaved function $f(x)$. If x has the dimension of momentum then

    (a). $\delta(x)$ has the dimension of momentum

    (b). $\delta(x)$ has the dimension of (momentum)^2

    (c). $\delta(x)$ is dimensionless

    (d). $\delta(x)$ has the dimension of (momentum)$^{-1}$

43. The operator A and B share all the eigenstates. Then the least possible value of the product of uncertainties $\Delta A \Delta B$ is

    (a). $\hbar$

    (b). 0

    (c). $\hbar/2$

    (d). Determinant(AB)

44. The resolving power of a grating spectrograph can be improved by

    (a). recording the spectrum in the lowest order

    (b). using a grating with longer grating period

    (c). masking a part of the grating surface

    (d). illuminating the grating to the maximum possible extent

45. The value of limit

$$\lim_{z \to i} \frac{z^{10} + 1}{z^6 + 1}$$

is equal to

    (a). 1

    (b). 0

    (c). $-10/3$

    (d). 5/3

46. A conducting sphere of radius r has charge Q on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will

    (a). increase four times

    (b). increase eight times

    (c). remain the same

    (d). decrease four times

47. Three sinusoidal waves have the same frequency with amplitude $A, A/2$ and $A/3$ while their phase angles are $0, \pi/2$ and π respectively. The amplitude of the resultant wave is

(a). $\frac{11A}{6}$

(b). $\frac{2A}{3}$

(c). $\frac{5A}{6}$

(d). $\frac{7A}{6}$

48. The value of integral

$$I = \oint_c \frac{\sin z}{2z - \pi} dz$$

with c a circle $|z| = 2$, is

(a). 0

(b). $2\pi i$

(c). πi

(d). $-\pi i$

49. Consider a square well of depth $-V_0$ and width a with $V_0 a$ fixed. Let $V_0 \to \infty$ and $a \to 0$. This potential well has

(a). No bound states

(b). 1 bound state

(c). 2 bound states

(d). Infinitely many bound states

50. If hydrogen atom is bombarded by energetic electrons, it will emit

(a). K_α X-rays

(b). β-rays

(c). neutrons

(d). none of the above

Answer Key

Answer JEST 2014 Key

Q.No	Ans Key	Q.No	Ans Key	Q.No	Ans Key
1	a	21	a	41	c
2	c	22	c	42	d
3	c	23	b	43	b
4	*	24	a	44	d
5	b	25	*	45	d
6	c	26	d	46	b
7	d	27	a	47	c
8	d	28	d	48	c
9	c	29	a	49	b
10	b	30	c	50	d
11	a	31	b		
12	b	32	a		
13	b	33	b		
14	a	34	c		
15	d	35	a		
16	b	36	b		
17	c	37	a		
18	b	38	a		
19	c	39	d		
20	a	40	b		