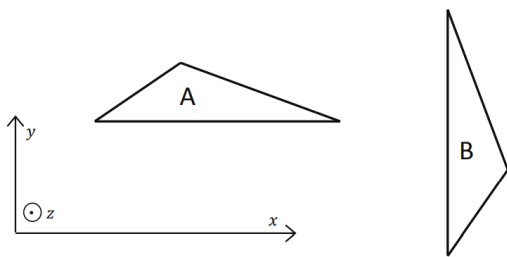


**TIFR- 2018**

1. Refer to the figure below. If the  $z$ -axis points out of the plane of the paper towards you, the triangle marked 'A' can be transformed (and suitably re-positioned) to the triangle marked 'B' by



- (a). rotation about  $x$ -direction by  $\pi/2$ , then rotation by  $-\pi/2$  in the  $yz$ -plane  
 (b). rotation about  $z$ -direction by  $\pi/2$ , then reflection in the  $yz$ -plane ✓  
 (c). reflection in the  $yz$ -plane, then rotation by  $\pi/2$  about  $z$ -direction  
 (d). reflection in the  $xz$ -plane, then rotation by  $-\pi/2$  about  $z$ -direction

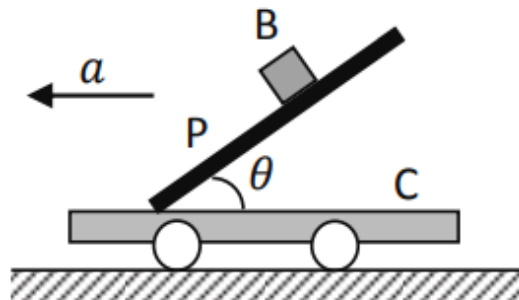
2. If a  $2 \times 2$  matrix  $\mathbb{M}$  is given by

$$\mathbb{M} = \begin{pmatrix} 1 & (1-i)/\sqrt{2} \\ (1+i)/\sqrt{2} & 0 \end{pmatrix}$$

then  $\det \exp \mathbb{M} =$

- (a).  $e$  ✓  
 (b).  $e^2$   
 (c).  $2i \sin \sqrt{2}$   
 (d).  $\exp(-2\sqrt{2})$

3. A small block B of mass  $m$  is quickly placed on an inclined plane P, which makes an angle  $\theta$  with a horizontal cart C, on which P is rigidly fixed (see figure). The coefficient of friction between the block B and the plane P is  $\mu$ . When the cart stays stationary the block slides down. If the cart C is moving in the horizontal direction with acceleration  $a$ , the minimum value of  $a$  for which the block will remain static is



- (a).  $g(\cos \theta - \mu \sin \theta)$  (b).  $g \frac{\tan \theta - \mu}{\mu \tan \theta + 1}$  ✓  
 (c).  $g \frac{1 - \mu \tan \theta}{\mu + \tan \theta}$  (d).  $g(\mu - \sin \theta \cos \theta)$

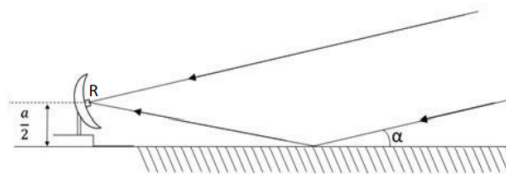
4. A particle of mass  $m$  moving in one-dimension  $x$  is subjected to the Lagrangian

$$L = \frac{1}{2} m (\dot{x} - \lambda x)^2$$

where  $\lambda$  is a real constant. If it starts at the origin at  $t = 0$ , its motion corresponds to the equation ( $a$  is a constant)

- (a).  $x = a \exp \lambda t$   $\exp(-\lambda t)$   
 (c).  $x = a \sin \lambda t$   
 (b).  $x = a\{1 - \exp(-\lambda t)\}$  (d).  $x = a \sinh \lambda t$  ✓

5. The sketch below shows a radio antenna located at the edge of a calm lake, which has a receiver  $R$  at the centre of the dish at a height  $a/2$  above the ground. This is picking up a signal from a distant radio-emitting star which is just rising above the horizon. However, the receiver also picks up a reflected signal from the surface of the lake, which, at the relevant radio-wavelength, may be taken to be a plane.



If the instantaneous angle of the star above the horizon is denoted  $\alpha$ , the receiver  $R$  will detect the first interference maximum when  $\alpha =$

- (a).  $\arcsin \left( \frac{\lambda}{a} \right)^{1/3}$   
 (b).  $\arcsin \left( \frac{\lambda}{2a} \right)^{1/3}$  ✓  
 (c).  $\arcsin \frac{\lambda}{a}$   
 (d).  $\arcsin \frac{\lambda}{2a}$

6. A particle is confined inside a one-dimensional box of length  $\ell$  and left undisturbed for a long time. In the most general case, its wave-function MUST be

- (a). the ground state of energy.  
(b). periodic, where  $\ell$  equals an integer number of periods.  
(c). a linear superposition of the energy eigenfunctions.✓  
(d). any one of the energy eigenfunctions.

7. A classical ideal gas of atoms with masses  $m$  is confined in a three-dimensional potential

$$V(x, y, z) = \frac{\lambda}{2}(x^2 + y^2 + z^2)$$

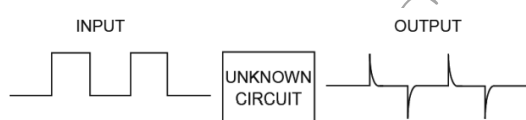
at a temperature  $T$ . If  $k_B$  is the Boltzmann constant, the root mean square (r.m.s.) distance of the atoms from the origin is

- (a).  $\left(\frac{3k_B T}{\lambda}\right)^{1/2}$ ✓  
(b).  $\left(\frac{3k_B T}{2\lambda}\right)^{1/2}$   
(c).  $\left(\frac{2k_B T}{3\lambda}\right)^{1/2}$   
(d).  $\left(\frac{k_B T}{\lambda}\right)^{1/2}$

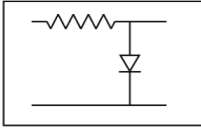
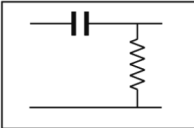
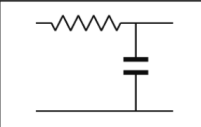
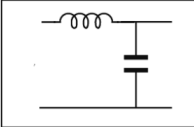
8. The characteristic impedance of a co-axial cable is independent of the

- (a). core diameter  
(b). outer diameter  
(c). length of the cable✓  
(d). dielectric medium between the core and the outer mesh

9. The figure below shows an unknown circuit, with an input and output voltage signal.



From the form of the input and output signals, one can infer that the circuit is likely to be

- (a)  (b) ✓   
(c)  (d) 

10. In Boolean terms,  $(A + B)(A + C)$  is equal to

- (a).  $ABC$  (b).  $(A + B + C)(A + B)$   
(c).  $A(B + C)$  (d).  $A + BC$ ✓

11. Consider the two equations

$$\begin{aligned} \frac{x^2}{3} + \frac{y^2}{2} &= 1 \\ x^3 - y &= 1 \end{aligned}$$

How many simultaneous real solutions does this pair of equations have?

Ans 002

12. A heavy steel ball is struck by a small steel pellet moving horizontally with velocity  $20 \text{ m s}^{-1}$ . If the pellet bounces off the steel ball with no slippage, and then rises vertically to a height  $10 \text{ m}$  above the point of contact, then what is 100 times the elastic coefficient of restitution ( $e$ ) i.e.  $100e$ ?

Ans 049

13. A particle is in the ground state of a cubical box of side  $\ell$ . Suddenly one side of the box changes from  $\ell$  to  $4\ell$ . If  $p$  is the probability of finding the particle in the ground state of the new box, what is  $1000p$ ?

Ans 058

14. The wave-function  $\Psi$  of a particle in a one-dimensional harmonic oscillator potential is given by

$$\Psi = \left(\frac{1}{\pi\ell^2}\right)^{1/4} \left(1 + \frac{\sqrt{2}x}{\ell}\right) \exp\left(-\frac{x^2}{2\ell^2}\right)$$

where  $\ell = 100 \mu\text{m}$ . Find the expectation value of the position  $x$  of this particle, in  $\mu\text{m}$ .

Ans 071

15. Consider a dipole antenna with length  $\ell$ , charge  $q$  and frequency  $\omega$ . The power emitted by the antenna at a large distance  $r$  is  $P$ . Now suppose the length  $\ell$  is increased to  $\sqrt{2}\ell$ , the charge is increased to  $\sqrt{3}q$  and the frequency is increased to  $\sqrt{5}\omega$ . By what factor is the radiated power increased?

Ans 150

16. Calculate the self-energy, in Joules, of a spherical conductor of radius  $8.5 \text{ cm}$ , which carries a charge  $100 \mu\text{C}$ .

Ans 529

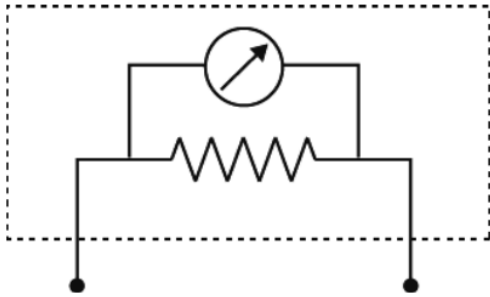
17. A heat engine is operated between two bodies that are kept at constant pressure. The constant-pressure heat capacity  $C_p$  of the reservoirs is independent of temperature. Initially the reservoirs are at temperature  $300 \text{ K}$  and  $402 \text{ K}$ . If, after some time, they come to a common final temperature  $T_f$ , the process remaining adiabatic, what is the value of  $T_f$  (in kelvin)?

Ans 347

18.  $N$  particles are distributed among three energy levels having energies:  $0, k_B T$  and  $2k_B T$  respectively. If the total equilibrium energy of the system is approximately  $42.5k_B T$  then find the value of  $N$  (to the closest integer).

Ans 100

19. A realistic voltmeter can be modelled as an ideal voltmeter with an input resistor in parallel as shown below:



Such a realistic voltmeter, with input resistance  $1\text{ k}\Omega$ , gives a reading of  $100\text{ mV}$  when connected to a voltage source with source resistance  $50\text{ }\Omega$ . What will a similar voltmeter, with input resistance  $1\text{ M}\Omega$ , read in  $\text{mV}$ , when connected to the same voltage source?

Ans 105

20. An electron enters a linear accelerator with a speed  $v = 10\text{ m}\cdot\text{s}^{-1}$ . A vertical section of the accelerator tube is shown in the figure, where the lengths of the successive sections are designed such that the electron takes the same time  $\tau = 20\text{ ns}$  to traverse each section.



If the momentum of the electron increases by  $2\%$  every time it crosses the narrow gap between two sections, what is the length (in  $\text{km}$ ) of the collider which will be required to accelerate it to  $100\text{ km}\cdot\text{s}^{-1}$ ?

Ans 102

21. If  $y(x)$  satisfies the differential equation

$$y'' - 4y' + 4y = 0$$

with boundary conditions  $y(0) = 1$  and  $y'(0) = 0$ , then  $y(-\frac{1}{2}) =$

- (a).  $\frac{2}{e}$  ✓  
 (b).  $\frac{1}{2} (e + \frac{1}{e})$   
 (c).  $\frac{1}{e}$   
 (d).  $-\frac{e}{2}$

22. Given the following  $xy$  data

$x$	1.0	2.0	3.0	4.0	5.0
$y$	0.002	0.601	0.948	1.21	1.42

which of the following would be the best curve, with constant positive parameters  $a$  and  $b$ , to fit this data?

- (a).  $y = ax - b$   
 (b).  $y = a + \exp bx$   
 (c).  $y = a \log_{10} bx$  ✓  
 (d).  $y = a - \exp(-bx)$

23. The Hamiltonian of a dynamical system is equal to its total energy, provided that its Lagrangian

- (a). does not contain velocity-dependent terms.  
 (b). is separable in generalized coordinates and velocities.  
 (c). does not have terms which explicitly depend on the coordinates.  
 (d). has no explicit time dependence. ✓

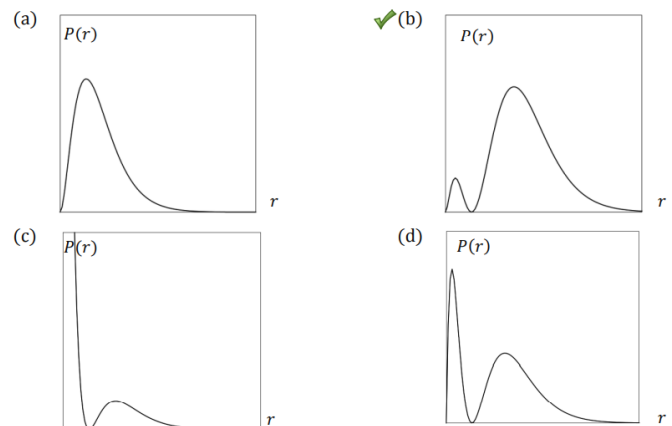
24. A particle in a one-dimensional harmonic oscillator potential is described by a wave-function  $\psi(x, t)$ . If the wavefunction changes to  $\psi(\lambda x, t)$  then the expectation value of kinetic energy  $T$  and the potential energy  $V$  will change, respectively, to

- (a).  $\lambda^2 T$  and  $V/\lambda^2$  ✓  
 (c).  $T/\lambda^2$  and  $\lambda^2 V$   
 (b).  $T/\lambda^2$  and  $V/\lambda^2$   
 (d).  $\lambda^2 T$  and  $\lambda^2 V$

25. An electron is in the  $2s$  level of the hydrogen atom, with the radial wave-function

$$\psi(r) = \frac{1}{2\sqrt{2}a_0^{3/2}} \left( 2 - \frac{r}{a_0} \right) \exp\left(-\frac{r}{2a_0}\right).$$

The probability  $P(r)$  of finding this electron between distances  $r$  to  $r + dr$  from the centre is best represented by the sketch



26. An atom of atomic number  $Z$  can be modelled as a point positive charge surrounded by a rigid uniformly negatively charged solid sphere of radius  $R$ . The electric polarisability  $\alpha$  of this system is defined as

$$\alpha = \frac{p_E}{E}$$

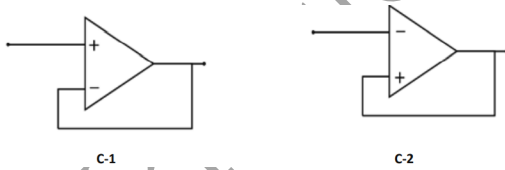
where  $p_E$  is the dipole moment induced on application of electric field  $E$  which is small compared to the binding electric field inside the atom. It follows that  $\alpha =$

- (a).  $\frac{8\pi\epsilon_0}{R^3}$   
 (b).  $\frac{4\pi\epsilon_0}{R^3}$   
 (c).  $8\pi\epsilon_0 R^3$   
 (d).  $4\pi\epsilon_0 R^3$  ✓

27. A many-body system undergoes a phase transition between two phases  $A$  and  $B$  at a temperature  $T_c$ . The temperature-dependent specific heat at constant volume  $C_V$  of the two phases are given by  $C_V^{(A)} = aT^3 + bT$  and  $C_V^{(B)} = cT^3$ . Assuming negligible volume change of the system, and no latent heat generated in the phase transition,  $T_c$  is

- (a).  $\sqrt{\frac{2b}{c}}$   
 (b).  $\sqrt{\frac{b}{c-a}}$   
 (c).  $\sqrt{\frac{3b}{c-a}}$  ✓  
 (d).  $\sqrt{\frac{4b}{c-a}}$

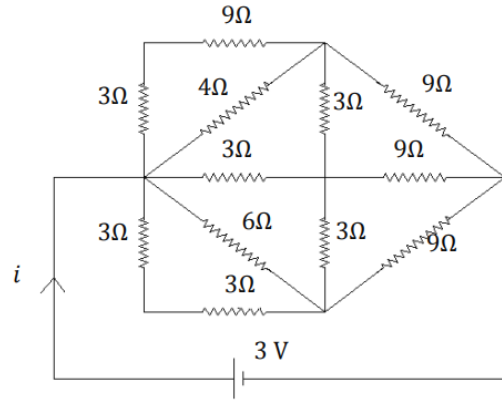
28. Consider the following circuits C-1 and C-2



You can apply the golden rules of an ideal op-amp to

- (a). only C-1 ✓ (b). only C-2  
 (c). both C-1 and C-2 (d). neither C-1 nor C-2

29. The current  $i$  flowing through the following circuit is



- (a). 0.5 A  
 (b). 0.6 A  
 (c). 0.75 A ✓  
 (d). 1.0 A

30. Two students A and B try to measure the time period  $T$  of a pendulum using the same stopwatch, but following two different methods. Student A measures the time taken for one oscillation, repeats this process  $N$  ( $\gg 1$ ) times and computes the average. On the other hand, Student B just once measures the time taken for  $N$  oscillations and divides that number by  $N$ .

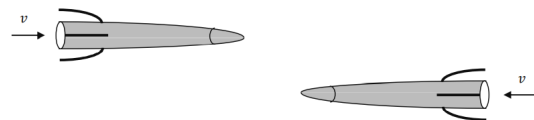
Which of the following statements is true about the errors in  $T$  as measured by A and by B?

- (a). The measurement made by A has a larger error than that made by B. ✓  
 (b). The measurement made by A has a smaller error than that made by B.  
 (c). A and B will measure the time period with the same accuracy.  
 (d). It is not possible to determine if the measurement made by A or B has the larger error.

31. Evaluate the integral

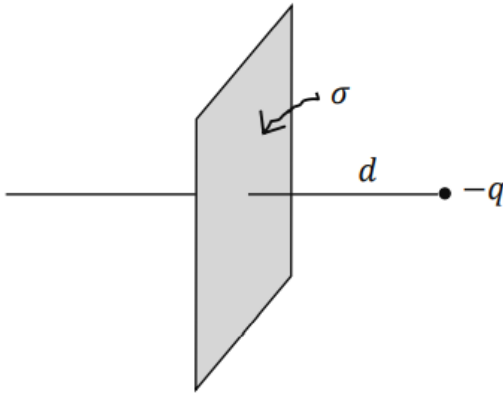
$$\int_{-\infty}^{\infty} dx \exp(-x^2) \cos(\sqrt{2}x)$$

32. From an observational post  $E$  on the Earth, two ballistic missiles, each of rest length  $\ell$  from nose-tip to tail-end, are observed to fly past each other, with the same uniform relativistic speed  $c/2$ , in opposite directions, as shown below.



What is the time taken for the tail-end of one of the missiles to cross the tail-end of the other missile, as measured from the post  $E$ ?

33. A statistical system, kept at a temperature  $T$ , has  $n$  discrete energy levels with equal level-spacing  $\varepsilon$ , starting from energy 0. If, now, a single particle is placed in the system what will be the mean energy of the system in the limit as  $n \rightarrow \infty$ ? [The answer should not be left as a summation]
34. Consider an infinite plane with a uniform positive charge density  $\sigma$  as shown below.



A negative point charge  $-q$  with mass  $m$  is held at rest at a distance  $d$  from the sheet and released. It will then undergo oscillatory motion. What is the time period of this oscillation?

[You may assume that the point charge can move freely though the charged plane without disturbing the charge density].

35. Given a particle confined in a one-dimensional box between  $x = -a$  and  $x = +a$ , a student attempts to find the ground state by assuming a wave-function

$$\psi(x) = \begin{cases} A(a^2 - x^2)^{3/2} & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$$

The ground state energy  $E_m$  is estimated by calculating the expectation value of energy with this trial wave-function. If  $E_0$  is the true ground state energy, what is the ratio  $E_m/E_0$ ?

36. The Fourier series which reproduces, in the interval  $0 \leq x < 1$ , the function

$$f(x) = \sum_{n=-\infty}^{+\infty} \delta(x - n)$$

where  $n$  is an integer, is

- (a).  $\cos \pi x + \cos 2\pi x + \cos 3\pi x + \dots$  (to  $\infty$ )  
 (b).  $1 + 2 \cos 2\pi x + 2 \cos 4\pi x + 2 \cos 6\pi x + \dots$  (to  $\infty$ ) ✓

- (c).  $1 + \cos \pi x + \cos 2\pi x + \cos 3\pi x + \dots$  (to  $\infty$ )

- (d).  $(\cos \pi x + \sin \pi x) + \frac{1}{2}(\cos 2\pi x + \sin 2\pi x) + \frac{1}{3}(\cos 3\pi x + \sin 3\pi x) + \dots$  (to  $\infty$ )

37. The value of the integral

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2}$$

is

- (a).  $1/2a$   
 (b).  $1/2\pi a$   
 (c).  $\pi a \exp(-a)$   
 (d).  $\exp(-a)/a$  ✓

38. A dynamical system with one degree of freedom is described by canonical coordinates  $(p, q)$ . The generator  $F$  of the canonical transformation  $(p, q) \rightarrow (-q, p)$  is

- (a).  $F = -p\dot{q}$  (b).  $F = p\dot{q}$   
 (c).  $F = -\dot{p}q$  (d).  $F = p\dot{q}$  ✓

39. The electrostatic charge density  $\rho(r)$  corresponding to the potential

$$\varphi(r) = \frac{q}{4\pi\epsilon_0} \frac{1}{r} \left(1 + \frac{\alpha r}{2}\right) \exp(-\alpha r)$$

is  $\rho =$

- (a).  $q\delta(r) - 2q\alpha^3 \exp(-\alpha r)$  (b).  $q\delta(r) - q\frac{\alpha^3}{4} \exp(-\alpha r)$   
 (c).  $q\delta(r) - q\frac{\alpha^3}{2} \exp(-\alpha r)$  ✓ (d).  $-q\delta(r) - 2q\alpha^3 \exp(-\alpha r)$

40. The Hamiltonian of a particle of charge  $q$  and mass  $m$  in an electromagnetic field is given by

$$H = \frac{1}{2m} |\vec{p} - q\vec{A}(\vec{x}, t)|^2 + q\varphi(\vec{x}, t)$$

where  $(\varphi, \vec{A})$  are the electromagnetic potentials. Clearly this Hamiltonian changes under a gauge transformation

$$\varphi \rightarrow \varphi - \frac{\partial \chi}{\partial t}, \vec{A} \rightarrow \vec{A} + \vec{\nabla} \chi$$

where  $\chi(\vec{x}, t)$  is a gauge function. Nevertheless the motion of the particle is not affected because

- (a). the Lagrangian does not change under the gauge transformation.  
 (b). the motion of the particle is correctly described only in the Lorenz gauge.  
 (c). the action of the particle changes only by surface terms which do not vary. ✓

(d). the Lorentz force is modified to balance the effect of the gauge transformation.

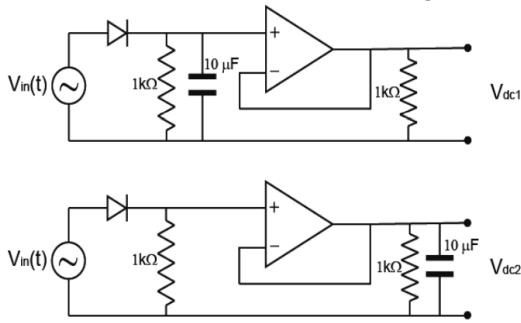
41. The electron of a free hydrogen atom is initially in a state with quantum numbers  $n = 3$  and  $\ell = 2$ . It then makes an electric dipole transition to a lower energy state. Which one of the given states could it be in after the transition?

- (a).  $n = 2, \ell = 1$  ✓ (b).  $n = 2, \ell = 2$   
(c).  $n = 3, \ell = 0$  (d).  $n = 3, \ell = 1$

42. Consider a monatomic solid lattice at a low temperature  $T \ll T_D$ , where  $T_D$  is the characteristic Debye temperature of the solid ( $T_D = \hbar\omega_m/k_B$  where  $\omega_m$  is the maximum possible frequency of the lattice vibrations). The heat capacity of the solid is proportional to

- (a).  $T/T_D$   
(b).  $(T/T_D)^3$  ✓  
(c).  $T_D/T$   
(d).  $(T_D/T)^2$

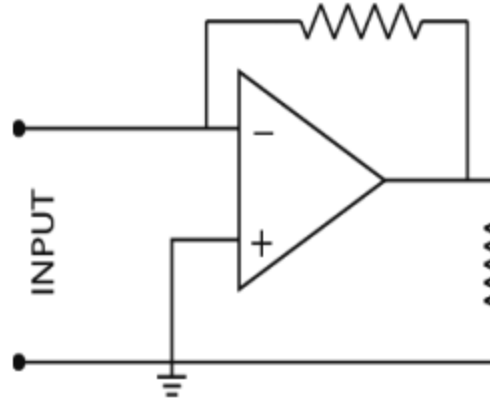
43. A signal  $V_{in}(t) = 5 \sin(100\pi t)$  is sent to both the circuits sketched below.



If the DC output voltage of the top circuit has a value  $V_{dc1}$  and the bottom circuit has a value  $V_{dc2}$ , then which of the following statements about the relative value of  $V_{dc1}$  and  $V_{dc2}$  is correct?

- (a).  $V_{dc1} < V_{dc2}$   
(b).  $V_{dc1} > V_{dc2}$  ✓  
(c).  $V_{dc1} = V_{dc2}$   
(d). It will depend on the slew rate of the op-amp.

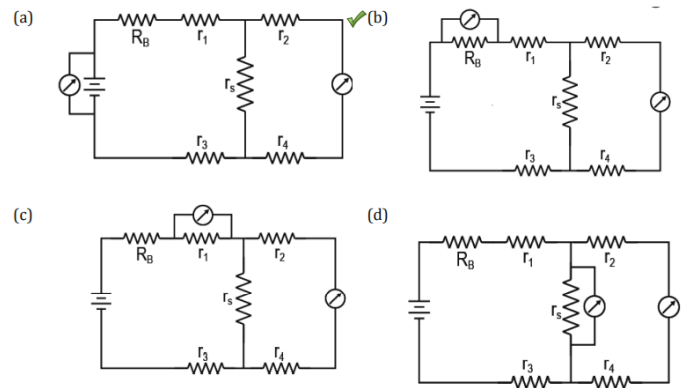
44. Consider the circuit shown on the right, which involves an op-amp and two resistors, with an input voltage marked INPUT.



Which of the following circuit components, when connected across the input terminals, is most likely to create a problem in the normal operation of the circuit?

- (a). A voltage source with very high Thevenin resistance.  
(b). A current source with a very high Norton resistance.  
(c). A voltage source with a very low Thevenin resistance. ✓  
(d). A current source with a very low Norton resistance.

45. Which one of the following circuits, constructed only with resistors and voltmeters, will allow you to obtain the correct value of resistance  $r_s$  using the voltmeter readings? Note that the value of  $R_B$  is known while  $r_1, r_2, r_3, r_4$  and  $r_5$  are all unknown. [Assume that the voltmeters and resistors are ideal.]



46. A fourth rank Cartesian tensor  $T_{ijkl}$  satisfies the following identities

- (i)  $T_{ijkl} = T_{jikl}$   
(ii)  $T_{ijkl} = T_{ijlk}$   
(iii)  $T_{ijkl} = T_{klij}$



Assuming a space of three dimensions (i.e.  $i, j, k = 1, 2, 3$ ), what is the number of independent components of  $T_{ijkl}$ ?

47. If the velocity of the Earth in its orbit is  $\nu$ , find  $\partial E/E$ , where  $E$  is the translational (non-relativistic) kinetic energy of the Earth and  $\delta E$  is its relativistic correction to the lowest order in  $\nu/c$ .
48. A plane electromagnetic wave, which has an electric field

$$\vec{E}(\vec{x}, t) = (P\hat{i} + Q\hat{j}) \exp i\omega \left( t - \frac{z}{c} \right)$$

is passing through vacuum. Here  $P, Q$  and  $\omega$  are all constants, while  $c$  is the speed of light in vacuo.

What is the average energy flux per unit time (in SI units) crossing a unit area placed normal to the direction of propagation of this wave, in terms of the above constants?

49. The state  $|\Psi\rangle$  of a spin-1 particle is given by

$$|\Psi\rangle = \frac{1}{\sqrt{3}} \left( |1, -1\rangle + |1, 0\rangle \exp \frac{i\pi}{3} + |1, 1\rangle \exp \frac{2i\pi}{3} \right)$$

where  $|S, M_S\rangle$  denote the spin eigenstates with eigenvalues  $\hbar^2 S(S+1)$  and  $\hbar M_S$  respectively. Find  $\langle S_x \rangle$ , i.e. the expectation value of the  $x$  component of the spin.

50. A particle of mass  $m$  moves in a two-dimensional space  $(x, y)$  under the influence of a Hamiltonian

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{4}m\omega^2(5x^2 + 5y^2 + 6xy)$$

Find the ground state energy of this particle in a quantum-mechanical treatment.

Answer Key:

Question No	Answer
1	b
2	a
3	b
4	d
5	d
6	c
7	a
8	c
9	b
10	d
11	002
12	049
13	058
14	071
15	150
16	529
17	347
18	100
19	105
20	102
21	a
22	c
23	d
24	a
25	b
26	d
27	c
28	a
29	c
30	a
31	-
32	-
33	-
34	-
35	-
36	b
37	d
38	d
39	c
40	c
41	a
42	b
43	b
44	c
45	a
46	-
47	-
48	-
49	-
50	-