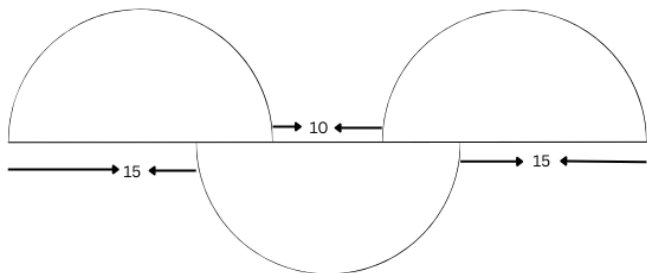


CSIR NET JUNE 2025**Section A: Qno.(1 to 20)**

1. Three identical semi-circles are arranged as shown. What is the diameter of the semi-circles?



1. 5π
 2. 20
 3. $15\pi/2$
 4. 25
2. Three friends, Mr. Rahman, Mr. George and Mr. Vedant, met after a long time. They were wearing red, green and violet colour shirts. Mr. Rahman and the person wearing violet shirt noticed that none of the three is wearing a colour that starts with same letter as his name. Which one of the following is the correct match of the persons with the colour of their shirts?

1. Rahman-Violet, George-Red, Vedant-Green
2. Rahman-Green, George-Violet, Vedant-Red
3. Rahman-Green, George-Red, Vedant-Violet
4. Rahman-Red, George-Violet, Vedant-Green

3. The value of a company is measured as the total value of its shares owned by different investors. Rakesh owns $\frac{2}{15}$ of the shares of a company. He sells $\frac{1}{3}$ of his shares for Rs. 75,000/-. What is the total value of the company at that time?

1. Rs. 15,75,800
2. Rs. 16,87,500
3. Rs. 17,75,800
4. Rs. 18,27,500

4. Rahul and his father started jogging on a circular track of radius r ($r > 2$). Rahul completed one round and stopped. His father got tired halfway into the first round and returned to his starting point along a straight line. What is the ratio of the distances covered by Rahul and his father?

1. $\pi/(\pi + 2)$

2. $2\pi/(\pi + 2)$

3. 1

4. 2

5. Kavita starts from her house and walks 200 m northward, then turns 45° right and walks 70 m. After that, she turns 90° right and walks 70 m. Which of the following is the closest value of the shortest distance between Kavita's current location and her house?

1. 296 m

2. 240 m

3. 200 m

4. 223 m

6. What will be the digit at the unit's place of $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$?

1. 0

2. 5

3. 7

4. 9

7. Numbers of Rose, Lotus, and Marigold plants in a garden are in the proportion 8:5:7. Later, 75%, 40% and 50% more plants of their respective categories were added. What will be the new proportion of plants, in the same order?

1. 5:3:4

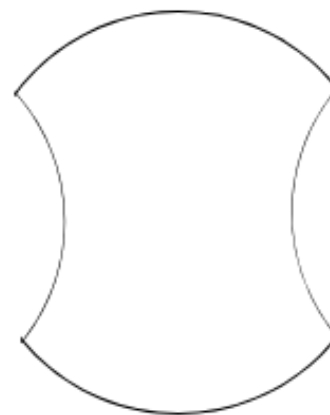
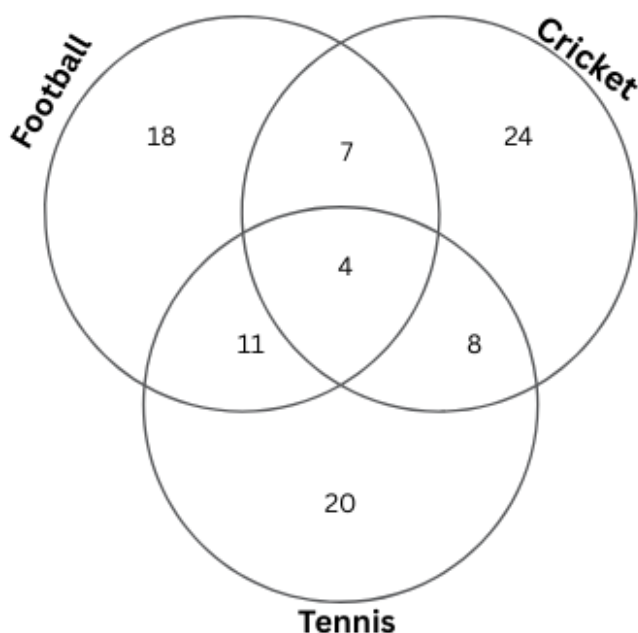
2. 4:2:3

3. 5:4:3

4. 7:4:5

8. The given Venn diagram shows numbers of players playing one or more than one sport.

The percentage of players who play exactly two sports is closest to ____%.



What is the area of this shape?

1. π
2. 1
3. 2
4. 4

1. 5
2. 14
3. 28
4. 32

9. In a code, the word DELTOID is written as 3152893. Then LOTION could be written as

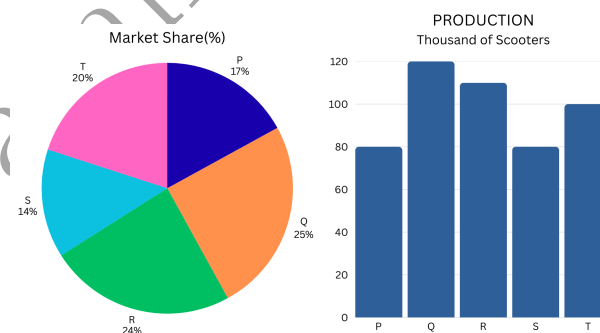
1. 582986
2. 582981
3. 198396
4. 198392

10. Suresh asked Ramesh to identify the person in a photo that the latter is holding. Ramesh responds, "I have no brothers or sisters. However, that man's father is my father's son." Who is the person in the photo?

1. Suresh
2. Ramesh
3. Ramesh's son
4. Ramesh's cousin

11. A circle of radius 1 unit is divided into four quarters and rejoined as shown below.

12. The market share (%) and annual production of scooters from five automobile companies P, Q, R, S, and T are shown in graphs.



If the profit of a company is directly proportional to the ratio of market share to production, then which of the following statements is/are CORRECT?

Statement X: Companies T and P have same profit
 Statement Y: Company R has the maximum profit
 Statement Z: Company S has the minimum profit

1. X and Y
2. X and Z
3. Y and Z
4. Only Z

13. Consider the following statements:

Statement I: All Booklets are Manuals.

Statement II: All Manuals are Catalogues.

2. 50%

If Statements I and II are True, which one of the following conclusions can be conclusively drawn?

3. 75%

4. 100%

1. All Manuals are Booklets.

2. All Catalogues are Booklets.

3. All Booklets are Catalogues.

4. All Catalogues are Manuals.

18. A car has wheels of diameter 36 cm. If it runs at a speed of 60 km/h, then the rotation per minute (RPM) will be closest to ____.

1. 884

2. 898

3. 906

4. 986

14. A cylindrical container of radius 20 cm was filled with water up to 25 cm height. A solid spherical ball of radius 7 cm was then immersed in the water. What would be the approximate increase in water level in the container after the ball was fully immersed?

1. 1.14 cm

2. 2.28 cm

3. 5.50 cm

4. 7.00 cm

19. Sum of the digits of a two-digit number 'ab' is subtracted from the number and the result is divided by 9. Then the result of this will be

1. always a

2. always b

3. neither a nor b

4. either a or b depending on $a + b$

15. A stock market trader has lost two thirds of her investment on a day. Next day she recovered one third of the previous day's loss. What fraction of her initial investment is she left with?

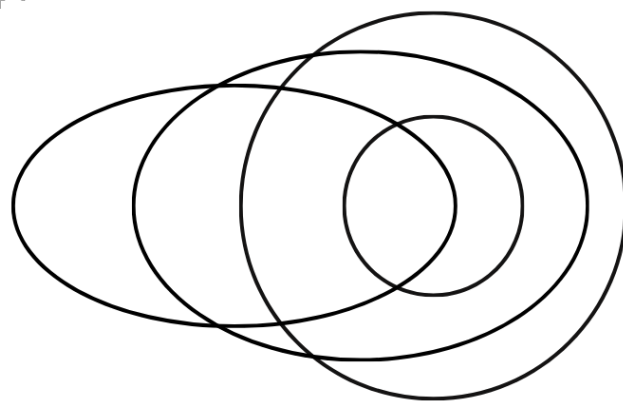
1. $\frac{1}{3}$

2. $\frac{2}{3}$

3. $\frac{2}{9}$

4. $\frac{5}{9}$

20. The following diagram represents the relationship between four categories.



The categories could be

1. Rivers, water bodies, oceans, sources of evaporation

2. Parliamentarians, celebrities, elected persons, professional politicians

3. Monkeys, four-legged animals, pet animals, land animals

4. Furniture, chairs, seats, wooden objects

16. The initial monthly salaries of employees John, Riya, and Sunil were in the proportion 4:3:5. After an increase of Rs 10000 monthly to all, the new proportion becomes 6:5:7. What was the initial salary of Sunil?

1. Rs 20000

2. Rs 25000

3. Rs 30000

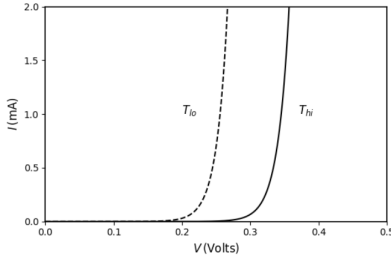
4. Rs 35000

17. A number is mistakenly divided by 2 instead of being multiplied by 2. What is the change in the result caused by this mistake?

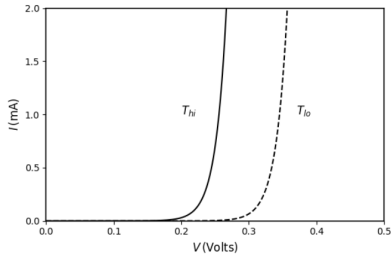
1. 25%

Section B: Qno.(21 to 45)

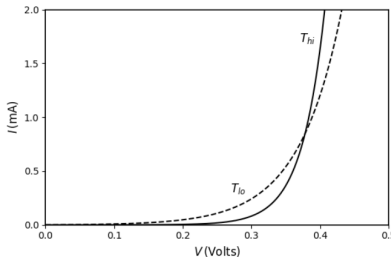
21. A Silicon p-n junction diode is operated under forward bias at two temperatures $T_{hi} \approx 300$ K (shown by solid line) and $T_{lo} \approx 200$ K (shown by dotted line). Which of the following plots best represents the I - V characteristics of the diode?



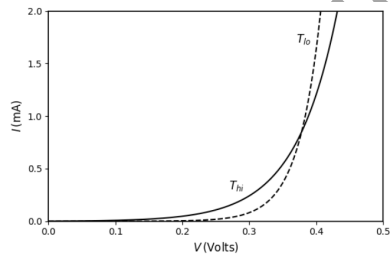
1.



2.

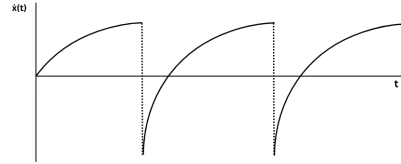


3.

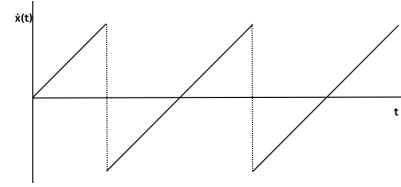


4.

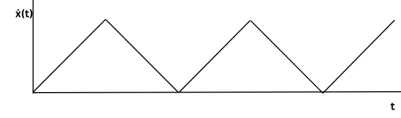
22. A particle of mass m is subjected to a potential $V(x) = V_0\theta(x) - kx$, where V_0 and k are positive constants and V_0 is much larger than the energy of the particle. The function $\theta(x) = 1$ for $x \geq 0$ and equals 0 otherwise. The particle starts from rest at $t = 0$, and $x = -5$. In the limit $V_0 \rightarrow \infty$, the graph for $\dot{x}(t)$ is best represented by



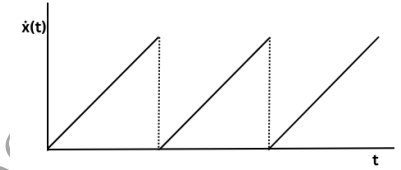
1.



2.

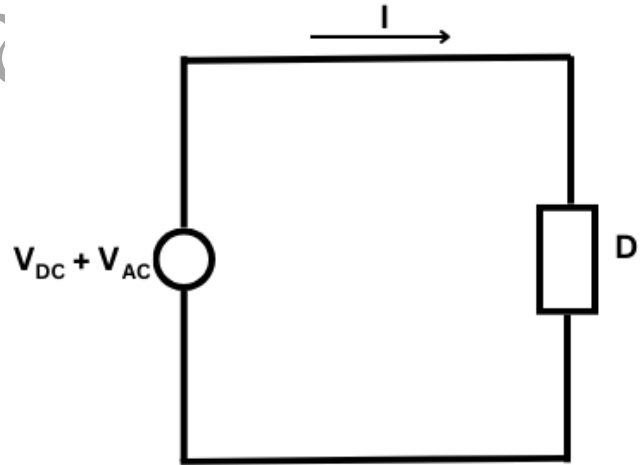


3.



4.

23. Consider the device D shown in the figure below. Its current-voltage characteristic is given by $I = aV + bV^2$, where I is the current, V is the input voltage, and a and b are constants. The device is used to mix a voltage signal $V = V_{DC} + V_{AC}$, where $V_{AC} = V_0 \cos \omega t$, V_{DC} and V_0 are constants.

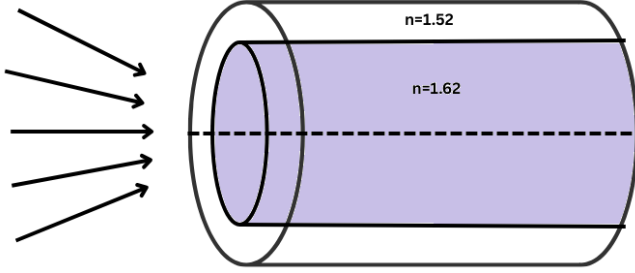


The frequency components present in the current I are

1. 0 and ω
2. 0, ω and 2ω
3. 0 and 2ω
4. ω and 2ω

24. A 1 km long optical fiber of core and clad refractive indices 1.62 and 1.52, respectively, is laid in a straight

line. Several identical light pulses are launched simultaneously from air on the entrance of this fiber from different angles about its axis, as shown below. The diameter of the fiber is small compared to its length. The maximum time difference between the pulses emerging at the other end of the fiber would be closest to



1. 355 ns
2. 317 ns
3. 5.40 μ s
4. 5.75 μ s

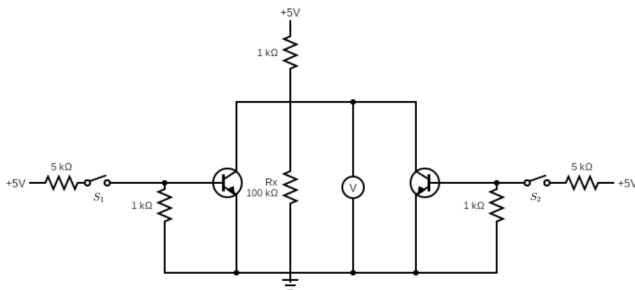
25. The energy eigenstates of a one-dimensional harmonic oscillator are denoted by $|i\rangle$, where $i = 0, 1, 2, 3, \dots$. If the momentum operator $\hat{p}$ satisfies

$$\frac{\langle n+1|\hat{p}|n\rangle}{\langle 2|p|1\rangle} = \sqrt{2}$$

then the value of n is

1. 0
2. 1
3. 2
4. 3

26. The circuit, composed of npn transistors of high β , resistors and switches, is shown in the figure. The biasing is sufficient to turn on the transistors when respective switches S1 and S2 are closed.



The voltage across the resistor $R_x = 100 \text{ k}\Omega$ is

1. $\sim 5 \text{ V}$ when both S1 and S2 are closed
2. $\sim 5 \text{ V}$ when either S1 or S2 are closed
3. $\sim 5 \text{ V}$ when both S1 and S2 are open
4. $\sim 0 \text{ V}$ when both S1 and S2 are open

27. A refrigerator can be thought to be a reversible engine operating between $T_2 = 20^\circ\text{C}$ and $T_1 = -10^\circ\text{C}$. The work needed to run this is supplied by another engine, that takes in energy at the rate of 500 W and runs with 50% efficiency. If the refrigerator freezes 5 kg of water at 0°C (latent heat $Q_L = 334 \text{ kJ/kg}$ for ice) in n hours, then n is closest to

1. 0.4
2. 0.3
3. 0.1
4. 0.2

28. A spin- $\frac{1}{2}$ system is prepared in the initial state $|\varphi\rangle = \frac{\sqrt{3}}{2}|+\rangle + \frac{1}{2}|-\rangle$ where $|+\rangle$ and $|-\rangle$ are eigenstates of $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively. A measurement of $\hat{S}_z$ is followed by a measurement of $\hat{S}_x$ on the system. What is the probability that the measurement of $\hat{S}_x$ yields a value $+\frac{\hbar}{2}$?

1. $\frac{1}{2}$
2. $\frac{2+\sqrt{3}}{4}$
3. $\frac{2-\sqrt{3}}{4}$
4. $\frac{3}{8}$

29. A plane electromagnetic wave $\vec{E}_1 \cos(k_z z + \omega t)$ is incident normally on a perfectly reflecting mirror in vacuum. If the permittivity of free space is ϵ_0 , the force exerted on an area A of the mirror would be

1. $A\epsilon_0|\vec{E}_1|^2\hat{z}$
2. $-\frac{A\epsilon_0}{2}|\vec{E}_1|^2\hat{z}$
3. $\frac{A\epsilon_0}{2}|\vec{E}_1|^2\hat{z}$
4. $-A\epsilon_0|\vec{E}_1|^2\hat{z}$

30. The Hamiltonian of a system is given by $H(x, p) = -[p^2 + V^2(x)]^{1/2}$, where x and p are generalized coordinate and momentum respectively and $V(x) \geq 0$. The corresponding Lagrangian is

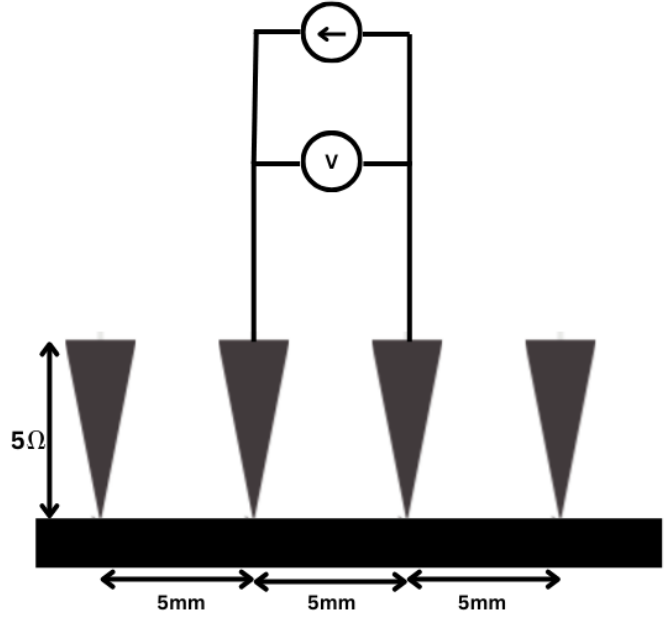
1. $-V(x)\sqrt{1-\dot{x}^2}$

2. $-V(x)/\sqrt{1-\dot{x}^2}$

3. $V(x)\sqrt{1-\dot{x}^2}$

4. $V(x)/\sqrt{1-\dot{x}^2}$

Configuration B



1. 1.1

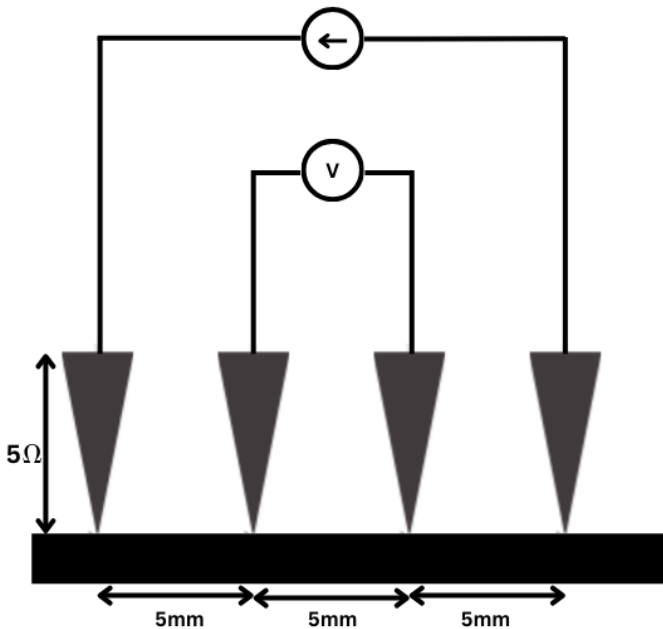
2. 1.2

3. 1.3

4. 1.5

31. Let R_A and R_B be the resistances of a channel determined (by taking the ratio of the voltage measured and current flowing) using configurations A and B respectively, as shown in the figure. In both configurations, each lead resistance is $5\ \Omega$ and each contact resistance is $10\ \Omega$. The channel has a resistivity of $20\ \Omega/\text{mm}$. Considering the voltmeter and the current source as ideal devices, the ratio R_B/R_A is:

Configuration A



1. $\frac{4ma^2}{3\pi\hbar}$

2. $\frac{4ma^2}{9\pi\hbar}$

3. $\frac{8ma^2}{3\pi\hbar}$

4. $\frac{8ma^2}{9\pi\hbar}$

32. A particle of mass m is in the third energy eigenstate of an infinite potential well of width a . The time interval in which the phase of this wave function changes by 2π is

33. For the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, which of the following is true?

1. $A^3 = 5A^2 - 4A - 2I$

2. $A^3 = 4A^2 - 6A + 3I$

3. $A^3 = 5A^2 - 5A - I$

4. $A^3 = 8A^2 + 3A - 4I$

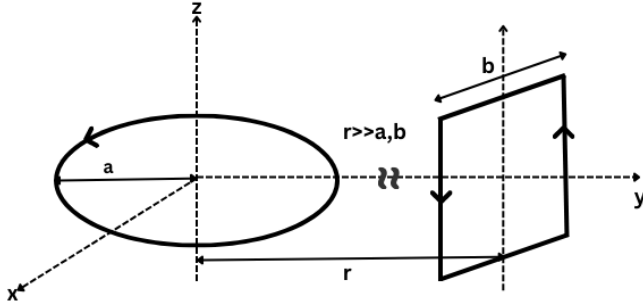
34. The solutions of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

1. ellipses with different eccentricities
2. circles with different centres
3. circles with different radii
4. ellipses with different foci

35. A circular loop of radius a (in the x - y plane) and a square loop of side b (in the x - z plane) are kept at a distance r . Both carry current I as shown in the figure. If $r \gg a, b$, the torque exerted on the square loop by the circular loop is



1. $-\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{z}$
2. 0
3. $\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{x}$
4. $-\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{x}$

36. Consider one mole of an ideal diatomic gas molecule at temperature T such that $k_B T \gg h\nu$, where ν is the frequency of its vibrational mode. If C_p and C_v are specific heats of this gas at constant pressure and volume respectively, then the ratio $\gamma = \frac{C_p}{C_v}$ is

1. 2
2. $\frac{7}{5}$
3. $\frac{5}{3}$
4. $\frac{9}{7}$

37. The value of the integral

$$\int_1^e dy \int_0^5 dx \delta(x^2 - y^2) \ln(xy)$$

is

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{1}{e}$
4. $\frac{e}{5}$

38. The Hamiltonian of the 1-dimensional quantum harmonic oscillator is given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. The expectation value of $[D, H]$ in the ground state, where $D = \frac{1}{2\hbar}(xp + px)$, is (in units of $\hbar\omega$):

1. i
2. $\frac{1}{2}$
3. $\frac{-3i}{2}$
4. 0

39. In a particular inertial frame, electric field $\vec{E}$ and magnetic field $\vec{B}$ are

$$\vec{E} = E_0 \vec{x}, \quad \vec{B} = \frac{E_0}{2c} \vec{x}.$$

Which of the following statements is true?

1. There exists an inertial frame where $\vec{E} = 0$, $\vec{B} \neq 0$
2. There exists no inertial frame where either $\vec{E} = 0$ or $\vec{B} = 0$
3. There exists an inertial frame where $\vec{B} = 0$, $\vec{E} \neq 0$
4. There exists an inertial frame where both $\vec{E} = 0$ and $\vec{B} = 0$

40. For the function $f(z) = \exp\left[z - 1 + \frac{1}{z-1}\right]$:

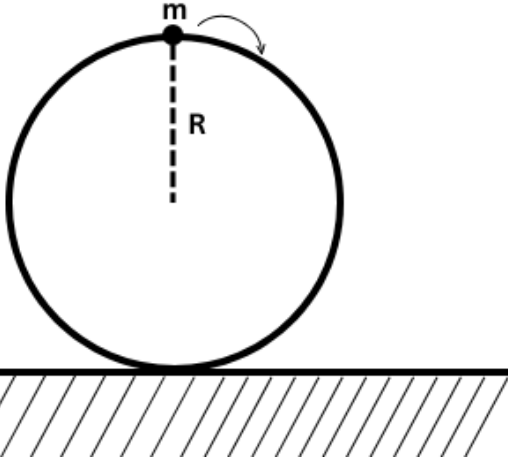
1. $z = 1$ is a pole of order one.
2. $z = 1$ is an essential singularity.
3. $z = 1$ is a pole of order two.
4. $z = 1$ is a removable singular point.

41. The internal energy of a system is given by $U = g(N)V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right]$, where V is the volume, S is the entropy, N is the number of molecules and R is a constant. The function $g(N)$ is proportional to

1. $N^{5/3}$
2. $N^{1/3}$
3. $N^{2/3}$

4. N

42. A sphere of radius R is held fixed on the horizontal ground. A point particle of mass m slides without friction from the top under the action of earth's gravity, as shown in the figure. The speed of the particle when it leaves the surface of the sphere is



1. $\sqrt{\frac{2}{3}gR}$

2. $\sqrt{\frac{3}{4}gR}$

3. $\sqrt{2gR}$

4. $\sqrt{gR}$

43. Consider the earth to be a free rigid body symmetric about its north-south (z) axis. If the principal moments of inertia satisfy $I_z = 1.003 I_x$, then its angular velocity (in the body fixed frame) would precess about the z -axis with a period of nearly

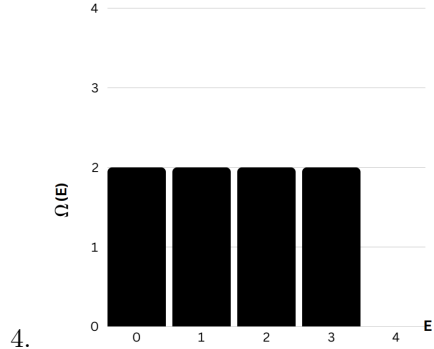
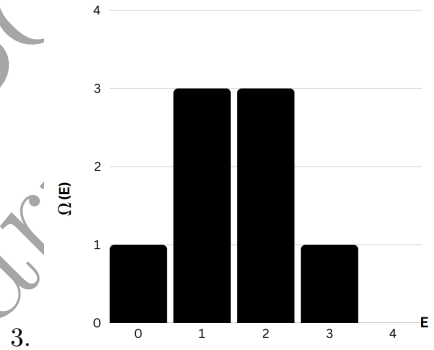
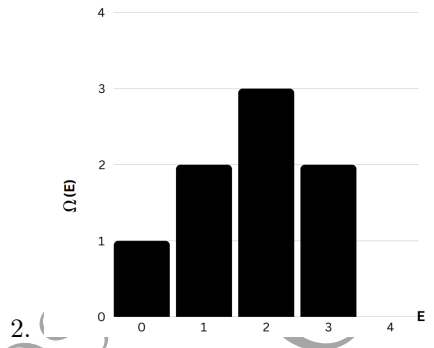
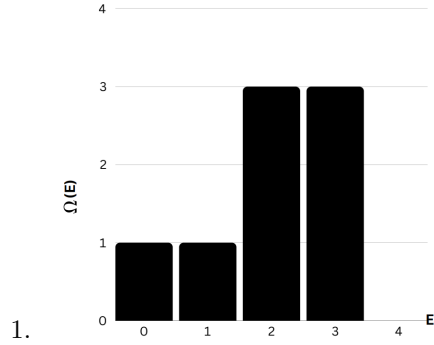
1. 167 days

2. 333 days

3. 556 days

4. 667 days

44. There are two boxes, one at the ground level, and the other at a fixed height h . There are three balls of different colours, each having mass m and radius $r \ll h$. There is no restriction on the number of balls that can be simultaneously put in a given box. For a given value of the total energy E (in units of mgh , g being the acceleration due to gravity), the number of accessible microstates is $\Omega(E)$. The plot of $\Omega(E)$ vs E is



45. A system consists of two non-interacting identical spin- $\frac{1}{2}$ particles. The spatial wave functions for the individual particles are given by $\varphi_1(x)$ and $\varphi_2(x)$. Let x_1 and x_2 denote the positions of the particles respectively. The total wave function of the system (not necessarily normalized) can be

1. $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)](|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)$

2. $[\varphi_1(x_1)\varphi_1(x_2) + \varphi_2(x_1)\varphi_2(x_2)]|\uparrow\rangle_1|\uparrow\rangle_2$

3. $\varphi_1(x_1)\varphi_2(x_2)|\uparrow\rangle_1|\uparrow\rangle_2$

4. $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2]$

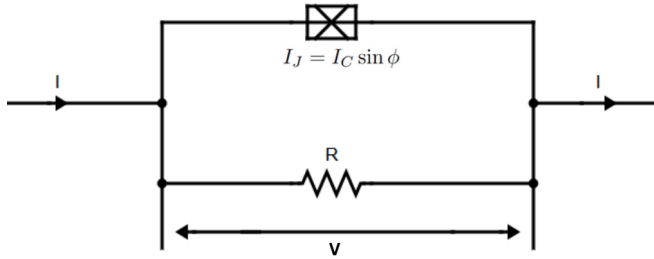
Section C: Qno.(46 to 75)

46. The current $I_J(t)$ through a Josephson junction (shown by the crossed box in the figure) and the voltage $V(t)$ across it, are given by

$$I_J(t) = I_C \sin \phi(t)$$

$$\frac{d\phi(t)}{dt} = \frac{2eV(t)}{\hbar}$$

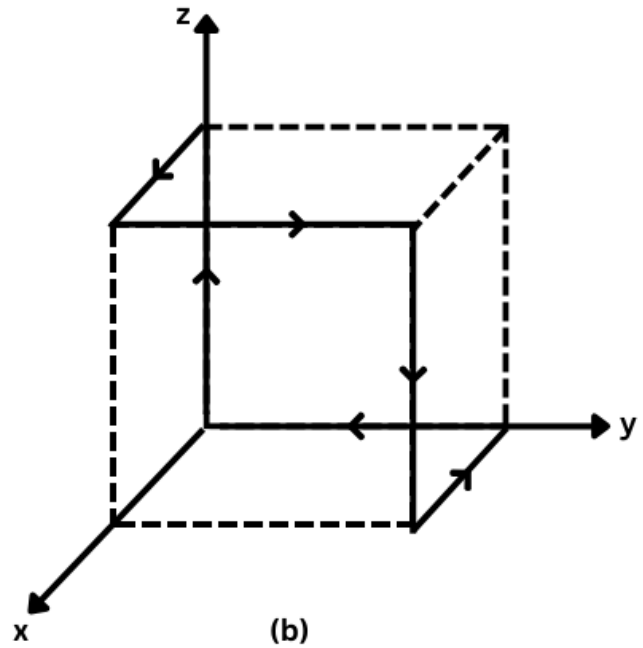
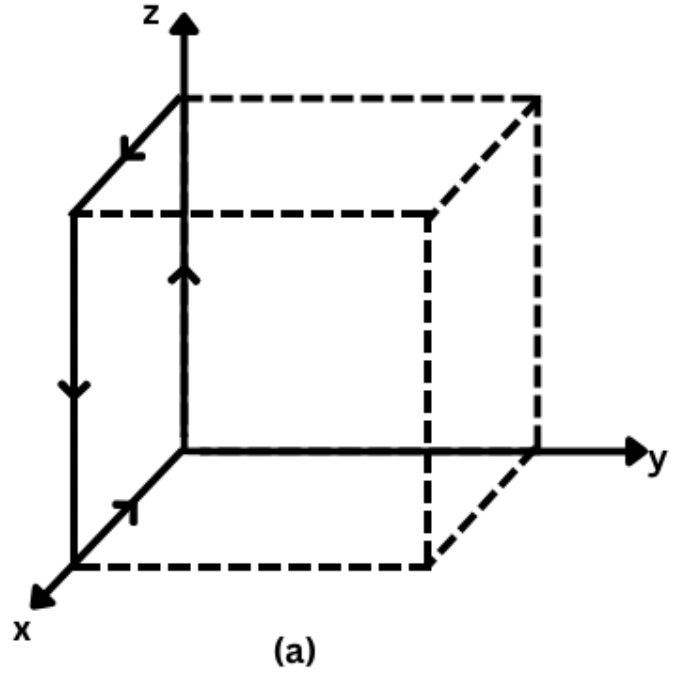
where I_C is the critical current of the junction and $\phi(t)$ is the phase difference across the junction. A resistor R is connected in parallel to the junction and a constant current $I > I_C$ flows through the combination as shown.



The energy dissipated in R in the time ϕ changes by 2π is

1. $\frac{\hbar}{2e} I$
2. $\frac{\hbar}{2e} I_C$
3. $\frac{\hbar}{2e} (I - I_C)$
4. $\frac{\hbar}{2e} (I + I_C)$

47. Two identical cubes are shown in figures (a) and (b). The magnitude of the magnetic field at the centre of the cube in (a), produced by the currents as shown, is B_0 . The magnitude of the magnetic field at the centre of the cube in (b) will be



1. $\sqrt{3}B_0$
2. $2B_0$
3. $\frac{3}{2}B_0$
4. $\sqrt{2}B_0$

48. In a rotational-vibrational spectrum of HCl (H^{35}Cl), the first R -branch line and the first P -branch line are observed at $\lambda^{-1} = 2906 \text{ cm}^{-1}$ and $\lambda^{-1} = 2865 \text{ cm}^{-1}$, respectively. The equilibrium bond length of this molecule would be closest to

1. $0.2 \text{ \AA}$
2. $1.3 \text{ \AA}$
3. $13 \text{ \AA}$
4. $2.1 \text{ \AA}$

49. The value of the integral

$$\int_0^\infty \frac{\cos ax}{1+x^2} dx$$

where a is a positive real number, is

1. $\frac{\pi}{2}e^{-a}$
2. πe^{-a}
3. $\frac{\pi}{2}e^{-(a/2)}$
4. $\pi e^{-(a/2)}$

50. For a free particle of mass m , consider the following time dependent quantity in phase space

$$Q = \frac{qp}{m} - \frac{p^2 t}{m^2},$$

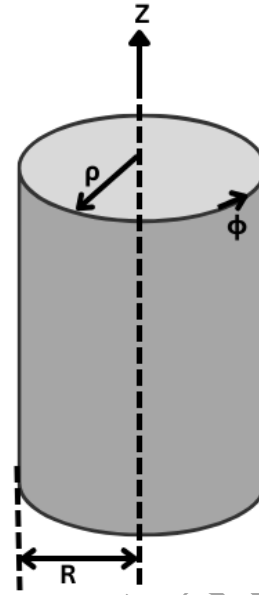
where q and p are the canonically conjugate position and momentum coordinates respectively. Then $\frac{dQ}{dt}$ is given by

1. 0
2. $\frac{p^2}{m^2}$
3. $-\frac{p^2}{m^2}$
4. $\frac{qp}{mt}$

51. From a straight-line segment of unit length, three points are chosen at random, one after another. The probability that they are in increasing order is

1. $\frac{1}{3}$
2. $\frac{1}{8}$
3. $\frac{1}{9}$
4. $\frac{1}{6}$

52. A long cylinder of radius R carries a magnetization $\vec{M} = k\rho^2\hat{\phi}$, where k is a constant, ρ is the radial distance from the axis and $\hat{\phi}$ is the azimuthal unit vector (see figure). The magnetic field inside and outside the cylinder would be



1. $\vec{B}_{\text{inside}} = 0, \vec{B}_{\text{outside}} = \mu_0 k \rho^2 \hat{\phi}$
2. $\vec{B}_{\text{inside}} = \mu_0 k \rho^2 \hat{\phi}, \vec{B}_{\text{outside}} = 0$
3. $\vec{B}_{\text{inside}} = \vec{B}_{\text{outside}} = \mu_0 k \rho^2 \hat{\phi}$
4. $\vec{B}_{\text{inside}} = \vec{B}_{\text{outside}} = 0$

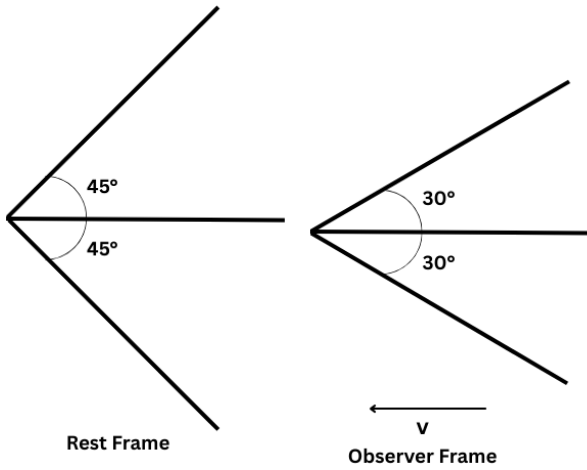
53. The ρ -mesons are $J^P = 1^-$ particles that decay strongly into pions. The ratio of the particle decay widths

$$\frac{\Gamma(\rho^0 \rightarrow \pi^0 \pi^0)}{\Gamma(\rho^+ \rightarrow \pi^+ \pi^0)}$$

is closest to

1. 1
2. $\frac{1}{2}$
3. 0
4. 2

54. In its rest frame, a source emits light in a conical beam of width -45° to 45° . An observer is moving towards the source with a speed v . For the observer, the beam width appears to be -30° to 30° . The speed of the observer is closest to



1. $0.62c$
2. $0.50c$
3. $0.82c$
4. $0.41c$

55. Let $P_n(x)$ be a polynomial of degree n with real coefficients, where $n = 0, 1, 2, 3, \dots$. If

$$\int_2^4 P_n(x)P_m(x)dx = \delta_{mn},$$

then

1. $P_1(x) = \pm\sqrt{\frac{3}{2}}(3-x)$
2. $P_1(x) = \pm\sqrt{\frac{3}{2}}(2-x)$
3. $P_1(x) = \pm\sqrt{\frac{3}{2}}(1-x)$
4. $P_1(x) = \pm\sqrt{3}(3+x)$

56. An atom is subjected to a weak magnetic field $B = 0.1$ T. A spectral line of wavelength 184.9 nm corresponding to a $J = 1$ to $J = 0$ transition splits into three components. The highest and the lowest components are separated by 3.2×10^{-4} nm. The magnetic moment of the atom in $J = 1$ state (in units of Bohr magneton) is

1. 2.82
2. 0.71
3. 1.41
4. 4.23

57. Consider $2N$ Ising spins, s_i ($s_i = \pm 1$), in a one-dimensional lattice with periodic boundary conditions. The Hamiltonian is given by

$$H = -J \sum_{i=1}^{2N} s_i s_{i+1}$$

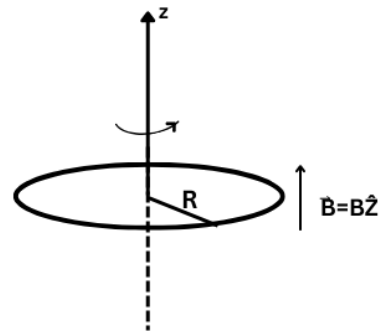
where J denotes the strength of the nearest-neighbour interactions with $J > 0$. Let F be the fully ferromagnetic state and let A be the lowest energy state with zero magnetization. The energy difference between these two states is

1. $\frac{3J}{2}$
2. $4J$
3. $\frac{J}{2}$
4. $2J$

58. Two discrete time random walkers start from the point $x = 0$ at time $t = 0$ taking discrete steps of unit length along the x axis. The first walker is unbiased and the second walker is biased to move towards the right with probability p . The probability that they are at a distance of 2 units from each other at both time steps $t = 1$ and $t = 2$ is

1. $\frac{1}{4}$
2. $\frac{1}{2} - \frac{p}{2}$
3. $1 - \frac{3p}{4}$
4. $\frac{p}{2}$

59. A thin circular wire loop of mass M , having radius R , carries a static charge Q . The plane of the loop is held perpendicular to a uniform magnetic field $\mathbf{B}$ along the z -axis passing through its centre, as shown in the figure. The loop, initially at rest, can freely rotate about the z -axis. When the magnetic field is switched off the loop starts rotating with an angular frequency



1. $\frac{QB}{M}$

2. $\frac{QB}{2M}$

3. $\frac{\pi QB}{M}$

4. $\frac{\pi QB}{2M}$

60. The minimum number of two input NOR gates required to obtain the following output for three digital inputs A, B and C

$$Y = (\overline{A} + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(A + \overline{B} + C)$$

would be

1. 4

2. 3

3. 5

4. 6

61. $|n\rangle$ denotes the eigenvector of the number operator for a particle of mass m in a one-dimensional potential $V = \frac{1}{2}m\omega^2 x^2$ ($n = 0, 1, 2, \dots$). For the state vector

$$|\varphi(x, t = 0)\rangle = \frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}|2\rangle$$

$\langle x(t) \rangle$ is

1. $\frac{2\sqrt{2}}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$

2. $\frac{4}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$

3. $\frac{2\sqrt{2}}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos 2\omega t$

4. $\frac{4}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos 2\omega t$

62. A thermodynamic system (at temperature T and volume V), is described by its internal energy $U = AT^4V$ and pressure $p = \frac{1}{3}AT^4$, where A is a constant of appropriate dimension. The Helmholtz free energy of the system is

1. $\frac{4}{3}AT^4V$

2. $\frac{1}{3}AT^4V$

3. $-\frac{1}{3}AT^4V$

4. $-\frac{4}{3}AT^4V$

63. A rigid molecule can have two possible rotational states: $j = 0$ or $j = 1$. Its rotational energies are given by $\epsilon_j = \frac{\hbar^2}{2I}j(j+1)$, where I is its moment of inertia. For an ensemble of such molecules in thermal equilibrium at temperature T , the ratio of the number of molecules in the $j = 1$ state (N_1), to those in $j = 0$ state (N_0), is $\frac{N_1}{N_0} = 0.003$. The temperature T (in units of $\frac{\hbar^2}{2Ik_B}$, where k_B is the Boltzmann constant) is closest to

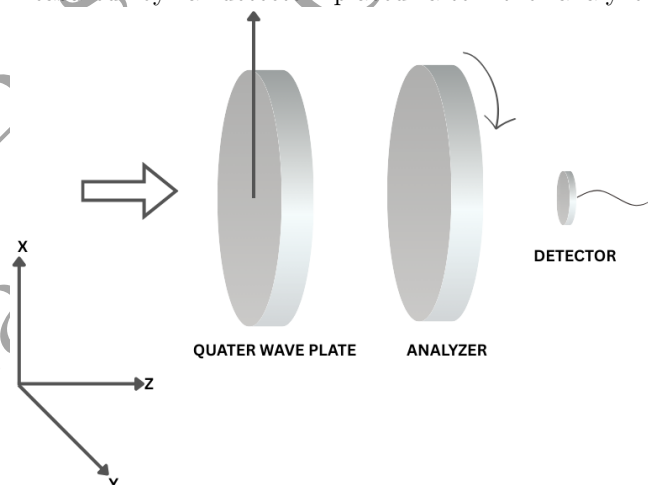
1. 0.29

2. 0.21

3. 0.15

4. 0.34

64. A beam of light along the z -axis passes through a quarter wave plate and an analyzer as shown in the figure. The fast axis of the quarter wave plate is aligned with the x -axis. The light intensity is measured by a detector placed after the analyzer.



Consider two scenarios where the incident light beam is (a) circularly polarized and (b) linearly polarized along the x -axis. If the polarization axis of the analyzer is rotated by one full cycle about the z -axis, the number of times the detector measures the maximum intensity in each case would be

1. (a) 4 and (b) 0

2. (a) 2 and (b) 0

3. (a) 4 and (b) 4

4. (a) 2 and (b) 2

65. The ground state wavefunction for the hydrogen atom is

$$\psi_0 = \sqrt{\frac{1}{\pi a_0^3}} e^{-r/a_0}$$

where a_0 is the Bohr radius.

Considering an additional potential H' as a perturbation to the hydrogen atom Hamiltonian, given by

$$H' = \left(\frac{e^2}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{R} \right] \right) \quad \text{for } 0 < r < R,$$

$$H' = 0 \quad \text{for } r > R,$$

where R is the radius of the proton, $R \ll a_0$. The shift in the ground state energy due to H' is

1. $\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{4R^2}{3a_0^2}$
2. $\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{R}{a_0}$
3. $-\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{2R^2}{a_0^2}$
4. $\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{2R^2}{3a_0^2}$

66. The probability density of a free particle of mass m at time $t = 0$, is given by

$$A \exp \left(-\frac{x^2}{2\sigma^2(0)} \right)$$

At $t > 0$, its probability density is proportional to

$$\exp \left(-\frac{x^2}{2\sigma^2(t)} \right)$$

where $\sigma^2(t)$ is

1. $\sigma^2(0) + \frac{\hbar^2 t^2}{\sigma^2(0)m^2}$
2. $\sigma^2(0) + \frac{\hbar^2 t^2}{4\sigma^2(0)m^2}$
3. $\sigma^2(0) + \frac{4\hbar^2 t^2}{\sigma^2(0)m^2}$
4. $\sigma^2(0) + \frac{2\hbar^2 t^2}{\sigma^2(0)m^2}$

67. A semiconductor has the dispersion relation $E = E_0 - A \cos(ak_x)$, where A and a are positive constants. The effective electron mass close to the minimum energy is

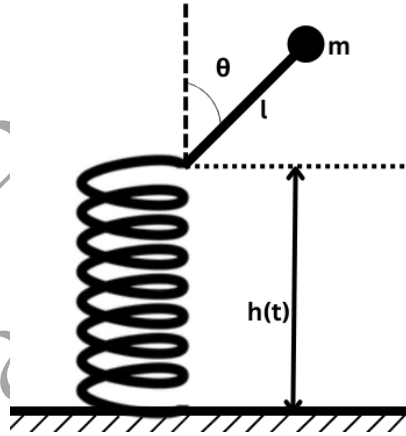
1. $\frac{\hbar^2}{A^2 a}$
2. $\frac{1}{4} \frac{\hbar^2}{A^2 a}$
3. $\frac{\hbar^2}{A a^2}$
4. $\frac{1}{2} \frac{\hbar^2}{A a^2}$

68. When a neutron of 1 keV kinetic energy impinges on a ^{12}C target, the total scattering cross section is 1000 barns. The approximate value of the phase shift δ_0 is

1. 18°
2. 108°
3. 90°
4. 36°

69. A massless rod of length l is hinged at the extreme end of a vertical spring whose other end is fixed to the ground. A point mass m is fixed at end of the rod, as shown in the figure.

Assume harmonic motion of the spring given by $h(t) = h_0(2 + \cos \omega t)$, where $h_0 > l$. The equation of motion of the mass (confined to the plane of the figure) is given by



1. $l\ddot{\theta} + \omega^2 h_0 \sin \theta \sin \omega t - g \sin \theta = 0$
2. $l\ddot{\theta} + \omega^2 h_0 \sin \theta \cos \omega t - g \sin \theta = 0$
3. $l\ddot{\theta} + \omega^2 h_0 \sin \theta \cos \omega t + g \sin \theta = 0$
4. $l\ddot{\theta} - \omega^2 h_0 \sin \theta \sin \omega t + g \sin \theta = 0$

70. A gas of electrons (with no source of scattering) is placed in an electric field

$$\vec{E} = E e^{i\omega t}(\hat{i} + \hat{k})$$

and a magnetic field

$$\vec{B} = B \hat{k}$$

where E and B are constants. The frequency at which the conductivity in the z -direction (given by the ratio of the current and the electric field, both in the z -direction) diverges is

1. 0

2. $\frac{eB}{m}$

3. $-\frac{eB}{m}$

4. $\frac{eB}{2m}$

71. A highly collimated laser beam with a diameter of 1 cm and wavelength 500 nm is directed from the earth's surface towards the moon ($\sim 384,000$ km away from the earth). Assuming ideal diffraction limited propagation in vacuum, which of the following best estimates the diameter of the beam upon returning to the earth after reflection from an ideal reflector installed on the moon.

1. 200 m

2. 20 m

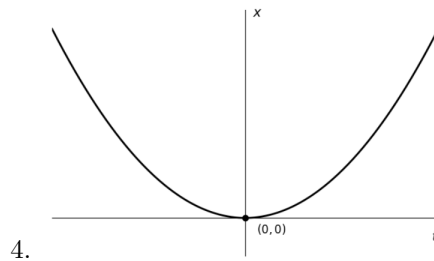
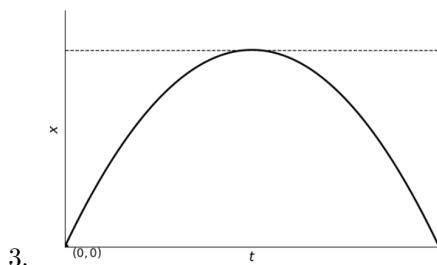
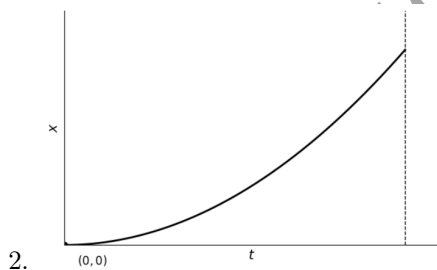
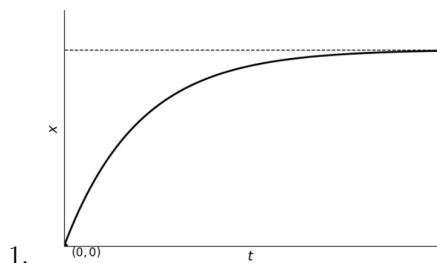
3. 20 km

4. 200 km

72. Which one of the following curves best represents the solution of the differential equation

$$\frac{dx}{dt} + x = 1,$$

with the initial condition $x(0) = 0$?



73. The charge density of the electron cloud of a hydrogen atom is given by

$$\rho(\vec{r}) = -\frac{e}{8\pi a^3} \exp\left(-\frac{r}{a}\right)$$

where a is some characteristic length. The potential energy due to the interaction between the proton (sitting at the origin) and the electron cloud is given by

1. $-\frac{e^2}{2\pi\epsilon_0 a}$

2. $-\frac{e^2}{4\pi\epsilon_0 a}$

3. $-\frac{e^2}{\pi\epsilon_0 a}$

4. $-\frac{e^2}{8\pi\epsilon_0 a}$

74. If the binding energies per nucleon of the nuclei $X(A=240)$ and $Y(A=120)$ are 7.6 MeV and 8.5 MeV respectively, the energy released in the symmetric fission, $X \rightarrow Y + Y$, is

1. 94 MeV

2. 9.4 MeV

3. 108 MeV

4. 216 MeV

75. Consider a laser cooling experiment where atoms are slowed down by an inelastic process of absorption and subsequent emission of photons. If light of wavelength 776.5 nm is used to slow down potassium atoms (mass number 39) with initial speed 130 ms^{-1} , the number of such absorption and emission cycles needed to bring the atoms to rest is closest to

1. 10^3

2. 10^2

3. 10^5

4. 10^4

ANSWER KEY

PART A		PART B		PART C	
1	2	21	2	46	1
2	2	22	2	47	1
3	2	23	2	48	2
4	2	24	1	49	1
5	4	25	4	50	1
6	2	26	3	51	4
7	2	27	4	52	2
8	3	28	1	53	3
9	1	29	4	54	4
10	3	30	3	55	1
11	3	31	3	56	3
12	3	32	2	57	2
13	3	33	1	58	1
14	1	34	3	59	2
15	4	35	4	60	1
16	2	36	4	61	2
17	3	37	1	62	3
18	1	38	4	63	1
19	1	39	2	64	4
20	4	40	2	65	4
		41	1	66	2
		42	1	67	3
		43	2	68	4
		44	3	69	2
		45	1	70	1
				71	3
				72	1
				73	4
				74	4
				75	4