3D and Angular Momentum CSIR

https://snboseplc.com/3d-and-angular-momentum-csir/

CSIR NET Questions

Q.No:1 CSIR-Dec-2014

Consider the normalized wavefunction

$$\phi = a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1-1}$$

where ψ_{lm} is a simultaneous normalized eigenfunction of the angular momentum operators L^2 and L_z, with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. If ϕ is an eigenfunction of the operator L_x with eigenvalue $\hbar$, then

1. $a_1 = -a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$
2. $a_1 = a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$
3. $a_1 = a_3 = \frac{1}{2}, a_2 = -\frac{1}{\sqrt{2}}$
4. $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

Q.No:2 CSIR-Dec-2014

Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two arbitrary constant vectors in three dimensions, the commutator $[\vec{\mathbf{a}}.\vec{\sigma}, \vec{\mathbf{b}}.\vec{\sigma}]$ is equal to (in the following I is the identity matrix)

1. $(\vec{\mathbf{a}}.\vec{\mathbf{b}})(\sigma_1 + \sigma_2 + \sigma_3)$
2. $2i(\vec{\mathbf{a}} \times \vec{\mathbf{b}}).\vec{\sigma}$
3. $(\vec{\mathbf{a}}.\vec{\mathbf{b}})I$
4. $|\vec{\mathbf{a}}||\vec{\mathbf{b}}|I$

Q.No:3 CSIR-June-2015

The ratio of the energy of the first excited state E_1, to that of the ground state E_o. of a particle in a three-dimensional rectangular box of sides L,L and L/2, is

1. 3:2
2. 2:1
3. 4:1
4. 4:3

Q.No:4 CSIR-June-2015

If L_i are the components of the angular momentum operator $\overrightarrow{L}$, then the arrow operator $\sum_{i=1,2,3}\left[\left[\vec{L}, L_i\right], L_i\right]$ equals

1. $\overrightarrow{L}$
2. $2\overrightarrow{L}$
3. $3\overrightarrow{L}$
4. $-\overrightarrow{L}$

Q.No:5 CSIR-Dec-2015

Let ψ_{nlm} denote the eigenstates of a hydrogen atom in the usual notation. The state

$$\frac{1}{5}[2\psi_{200} - 3\psi_{211} + \sqrt{7}\psi_{210} - \sqrt{5}\psi_{21-1}]$$

is an eigenstate of

1. L^2, but not of the Hamiltonian or L_z
2. the Hamiltonian, but not of L^2 or L_z
3. the Hamiltonian, L^2 and L_z
4. L^2 and L_z, but not of the Hamiltonian

Q.No:6 CSIR-Dec-2015

The Hamiltonian for a spin-1/2 particle at rest is given by $H = E_0(\sigma_z + \alpha\sigma_x)$, where σ_x and σ_z are Pauli spin matrices and E_0 and α are constants. The eigenvalues of this Hamiltonian are

1. $\pm E_0\sqrt{1+\alpha^2}$
2. $\pm E_0\sqrt{1-\alpha^2}$
3. E_0 (doubly degenerate)
4. $E_0\left(1 \pm \frac{1}{2}\alpha^2\right)$

Q.No:7 CSIR-Dec-2015

The product of the uncertainties $(\Delta L_x)(\Delta L_y)$ for a particle in the state $a|1,1\rangle + b|1,-1\rangle$ (where $|l,m\rangle$ denotes an eigenstate of L^2 and L_z) will be a minimum for

1. $a = \pm ib$
2. $a = 0$ and $b = 1$
3. $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$
4. $a = \pm b$

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Q.No:8 CSIR-June-2016

If $\hat{L}_x, \hat{L}_y$ and $\hat{L}_z$ are the components of the angular momentum operator in three dimensions, the commutator $[\hat{L}_x, \hat{L}_x\hat{L}_y\hat{L}_z]$ may be simplified to

1. $i\hbar L_x(\hat{L}_z^2 - \hat{L}_y^2)$
2. $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$
3. $i\hbar L_x(2\hat{L}_z^2 - \hat{L}_y^2)$
4. $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$

Q.No:9 CSIR-June-2016

Suppose that the Coulomb potential of the hydrogen atom is changed by adding an inverse-square term such that the total potential is $V(\vec{r}) = -\frac{Ze^2}{r} + \frac{g}{r^2}$, where g is a constant. The energy eigenvalues E_{nlm} in the modified potential

1. depend on n and l, but not on m
2. depend on n but not on l and m
3. depend on n and m, but not on l
4. depend explicitly on all three quantum numbers n, l and m

Q.No:10 CSIR-June-2017

Let $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and $x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(\frac{i\theta\sigma_z}{2}\right) \times [x\sigma_x + y\sigma_y + z\sigma_z] \exp\left(-\frac{i\theta\sigma_z}{2}\right)$. Then the coordinates are related as follows

1. $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
2. $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
3. $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
4. $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Q.No:11 CSIR-Dec-2017

The normalized wavefunction of a particle in three dimensions is given by $\psi(r, \theta, \varphi) = \frac{1}{\sqrt{8\pi a^3}} e^{-r/2a}$, where $a > 0$ is a constant. The ratio of the most probable distance from the origin to the mean distance from the origin, is. [You may use $\int_0^\infty dx x^n e^{-x} = n!$.]

1. 1/3
2. 1/2
3. 3/2
4. 2/3

Q.No:12 CSIR-June-2018

A particle of mass m is confined in a three-dimensional box by the potential

$$V(x, y, z) = \begin{cases} 0, & 0 \le x, y, z \le a, \\ \infty & \text{otherwise.} \end{cases}$$

The number of eigenstates of Hamiltonian with energy $\frac{9\hbar^2\pi^2}{2ma^2}$ is

1. 1
2. 6
3. 3
4. 4

Q.No:13 CSIR-June-2018

The Hamiltonian of a spin-$\frac{1}{2}$ particle in a magnetic field $\vec{B}$ is given by $H = -\mu\vec{B}.\vec{\sigma}$, where μ is a real constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. If $\vec{B} = (B_0, B_0, 0)$ and the spin state at time $t = 0$ is an eigenstate of σ_x, then of the expectation values $\langle\sigma_x\rangle, \langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$

1. only $\langle\sigma_x\rangle$ changes with time
2. only $\langle\sigma_y\rangle$ changes with time
3. only $\langle\sigma_z\rangle$ changes with time
4. all three change with time

Q.No:14 CSIR-June-2018

Two Stern-Gerlach apparatus S_1 and S_2 are kept in a line (x-axis). The directions of their magnetic fields are along the positive z- and y-axes, respectively. Each apparatus only transmits particles with spins aligned in the direction of its magnetic field. If an initially unpolarized beam of spin-$\frac{1}{2}$ particles passes through this configuration, the ratio of intensities $I_0 : I_f$ of the initial and final beams, is

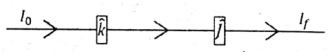


Figure 1: Stern-Gerlach setup diagram

1. 16 : 1

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2. 2 : 1
3. 4 : 1
4. 1 : 0

Q.No:15 CSIR-Dec-2018

The ground state energy of an anisotropic harmonic oscillator described by the potential $V(x, y, z) = \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2 + 8m\omega^2z^2$ (in units of $\hbar\omega$) is

1. 5/2
2. 7/2
3. 3/2
4. 1/2

Q.No:16 CSIR-Dec-2018

Let the wavefunction of the electron in a hydrogen atom be

$$\psi(\vec{r}) = \frac{1}{\sqrt{6}}\phi_{200}(\vec{r}) + \sqrt{\frac{2}{3}}\phi_{21-1}(\vec{r}) - \frac{1}{\sqrt{6}}\phi_{100}(\vec{r})$$

where ϕ_{nlm} are the eigenstates of the Hamiltonian in the standard notation. The expectation value of the energy in this state is

1. $-10.8eV$
2. $-6.2eV$
3. $-9.5eV$
4. $-5.1eV$

Q.No:17 CSIR-Dec-2018

Consider the operator $A_x = L_yp_z - L_zp_y$, where L_i and p_i denote, respectively, the components of the angular momentum and momentum operators. The commutator $[A_x, x]$, where x is the x-component of the position operator, is

1. $-i\hbar(zp_z + yp_y)$
2. $-i\hbar(zp_z - yp_y)$
3. $i\hbar(zp_z + yp_y)$
4. $i\hbar(zp_z - yp_y)$

Q.No:18 CSIR-Dec-2018

If the position of the electron in the ground state of a Hydrogen atom is measured, the probability that it will be found at a distance $r \geq a_0$ (a_0 being Bohr radius) is nearest to

1. 0.91
2. 0.66
3. 0.32
4. 0.13

Q.No:19 CSIR-Dec-2018

A system of spin-$\frac{1}{2}$ particles is prepared to be in the eigenstate of σ_z with eigenvalue +1. The system is rotated by an angle of 60° about the x-axis. After the rotation, the fraction of the particles that will be measured to be in the eigenstate of σ_z with eigenvalue +1 is

1. 1/3
2. 2/3
3. 1/4
4. 3/4

Q.No:20 CSIR-June-2019

A particle moving in a central potential is described by a wavefunction $\psi(\mathbf{r}) = zf(r)$ where $\mathbf{r} = (x, y, z)$ is the position vector of the particle and $f(r)$ is a function of $r = |\mathbf{r}|$. If $\mathbf{L}$ is the total angular momentum of the particle, the value of L^2 must be

1. $2\hbar^2$
2. $\hbar^2$
3. $4\hbar^2$
4. $\frac{3}{4}\hbar^2$

Q.No:21 CSIR-Dec-2019

The normalized wavefunction of a particle in three dimensions is given by $\psi(x, y, z) = Nz\exp[-a(x^2 + y^2 + z^2)]$ where a is a positive constant and N is a normalization constant. If $\mathbf{L}$ is the angular momentum operator, the eigenvalues of $\mathbf{L}^2$ and L_z, respectively, are

1. $2\hbar^2$ and $\hbar$
2. $\hbar^2$ and 0
3. $2\hbar^2$ and 0
4. $\frac{3}{4}\hbar^2$ and $\frac{1}{2}\hbar$

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Q.No:22 CSIR-Dec-2019

The wavefunction of a particle of mass m, constrained to move on a circle of unit radius centered at the origin in the xy-plane, is described by $\psi(\phi) = A\cos^2\phi$, where ϕ is the aziamuthal angle. All the possible outcomes of measurements of the z-component of the angular momentum L_z in this state, in units of $\hbar$, are

1. ± 1 and 0
2. ± 1
3. ± 2
4. ± 2 and 0

Q.No:23 Assam CSIR-Dec-2019

The Hamiltonian of a spin $\frac{1}{2}$ particle, precessing in the presence of a magnetic field B along the positive x-direction, is $H = -\mu_B B S_x$, where μ_B is the Bohr magneton. At time $t = 0$, the particle is in an eigenstate of S_z with eigenvalue $\hbar/2$. The earliest time when it will be in the eigenstate of S_y with eigenvalue $\hbar/2$, is

1. $\pi/(2\mu_B B)$
2. $\pi/(4\mu_B B)$
3. $3\pi/(2\mu_B B)$
4. $2\pi/(3\mu_B B)$

Q.No:24 CSIR-June-2020

An angular momentum eigenstate $|j, 0\rangle$ is rotated by an infinitesimally small angle ε about the positive y-axis in the counter clockwise direction. The rotated state, to order ε (upto a normalisation constant), is

1. (a) $|j,0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}(|j,1\rangle + |j,-1\rangle)$
2. (b) $|j,0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}(|j,1\rangle - |j,-1\rangle)$
3. (c) $|j,0\rangle - \frac{\varepsilon}{2}\sqrt{j(j-1)}(|j,1\rangle - |j,-1\rangle)$
4. (d) $|j,0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}|j,1\rangle - \frac{\varepsilon}{2}\sqrt{j(j-1)}|j,-1\rangle$

Q.No:25 CSIR-June-2020

The state of an electron in a hydrogen atom is

$$|\psi\rangle = \frac{1}{\sqrt{6}}|1,0,0\rangle + \frac{1}{\sqrt{3}}|2,1,0\rangle + \frac{1}{\sqrt{2}}|3,1,-1\rangle$$

where $|n,l,m\rangle$ denotes common eigenstates of $\hat{H}, \hat{L}^2$ and $\hat{L}_z$ operators in the standard notation. In a measurement of $\hat{L}_z$ for the electron in this state, the result is recorded to be 0. Subsequently a measurement of energy is performed. The probability that the result is E_2 (the energy of the $n = 2$ state) is

1. (a) 1
2. (b) 1/2
3. (c) 2/3
4. (d) 1/3

Q.No:26 CSIR-Feb-2022

Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions?

1. square of the radial position and z -component of angular momentum (r^2 and L_z)
2. x - components of linear and angular momenta (p_x and L_x)
3. y -component of position and z -component of angular momentum (y and L^2)
4. squares of the magnitudes of the linear and angular momenta (p^2 and L^2)

Q.No:27 CSIR-Sep-2022

Consider the Hamiltonian $H = AI + B\sigma_x + C\sigma_y$, where A, B and C are positive constants, I is the 2×2 matrix and σ_x and σ_y are Pouli matrices. If the normalized eigenvector corresponding to its largest energy eigenvelue is $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ y\end{pmatrix}$, then y is

1. $\frac{B+iC}{\sqrt{B^2+C^2}}$
2. $\frac{A-iB}{\sqrt{A^2+B^2}}$
3. $\frac{A-iC}{\sqrt{A^2+C^2}}$
4. $\frac{B-iC}{\sqrt{B^2+C^2}}$

Q.No:28 CSIR-Sep-2022

The momentum space representation of the Schrodinger equation of a particle in a potential $V(\vec{r})$ is

$$\left(|\mathbf{p}|^2 + \beta(\nabla_p^2)^2\right)\psi(p,t) = i\hbar\frac{\partial}{\partial t}\psi(p,t)$$

where $(\nabla_p)_i = \frac{\partial}{\partial p_i}$, and β is a constant. The potential is (in the following V_0 and a are constants.)

1. $V_0 e^{-r^2/a^2}$
2. $V_0 e^{-r^4/a^4}$
3. $V_0(\frac{r}{a})^2$
4. $V_0(\frac{r}{a})^4$

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Q.No:29 CSIR-Sep-2022

The Hamiltonian of a spin-1/2 particle in a magnetic field $B = B_0\hat{k}$ is given by $H = \lambda S \cdot B$, where S is its spin (in units of $\hbar$) and λ is constant. If the average spin density is $\langle S\rangle$ for an ensemble of such non-interacting particles, then $\frac{d}{dt}\langle S_x\rangle$ is

1. $\frac{\lambda}{\hbar}B_0\langle S_x\rangle$
2. $\frac{\lambda}{\hbar}B_0\langle S_y\rangle$
3. $-\frac{\lambda}{\hbar}B_0\langle S_x\rangle$
4. $-\frac{\lambda}{\hbar}B_0\langle S_y\rangle$

Q.No: 30 CSIR-Dec-2023

The Schrödinger wave function for a stationary state of an atom in spherical polar coordinates (r, θ, ϕ) is

$$\psi = Af(r)\sin\theta\cos\theta\, e^{i\phi}$$

where A is the normalization constant. The eigenvalue of $\hat{L}_z$ for this state is

1. $2\hbar$
2. $\hbar$
3. $-2\hbar$
4. $-\hbar$

Q.No: 31 CSIR-Dec-2023

The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$, is

$$\hat{H} = \frac{\epsilon}{\hbar^2}\left[(\hat{L}_1 + \hat{L}_2)\cdot\hat{L}_2 - (\hat{L}_{1z} + \hat{L}_{2z})^2\right].$$

If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$) are

1. $3\epsilon, 2\epsilon, -\epsilon$
2. $6\epsilon, 5\epsilon, 2\epsilon$
3. $3\epsilon, 2\epsilon, \epsilon$
4. $-3\epsilon, -2\epsilon, \epsilon$

Q.No: 32 CSIR-Dec-2023

The normalized wave function of an electron is

$$\psi(\vec{r}) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\phi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\phi)\chi_+\right],$$

where Y_l^m are the normalized spherical harmonics and $\chi_\pm$ denote the wavefunction for the two spin states with eigenvalues $\pm\frac{1}{2}\hbar$. The expectation value of the z component of the total angular momentum in the above state is

1. $-\frac{3}{4}\hbar$
2. $-\frac{9}{8}\hbar$
3. $\frac{3}{4}\hbar$
4. $\frac{9}{8}\hbar$

Q.No: 33 CSIR-Dec-2023

A quantum system is described by the Hamiltonian

$$H = -J\sigma_z + \lambda(t)\sigma_x,$$

where σ_i $(i = x, y, z)$ are Pauli matrices, J and λ are positive constants $(J \gg \lambda)$ and

$$\lambda(t) = \begin{cases} 0 & \text{for } t < 0 \\ \lambda & \text{for } 0 < t < T \end{cases}$$

At $t < 0$, the system is in the ground state. The probability of finding the system in the excited state at $t \gg T$, in the leading order in λ is

1. $\frac{\lambda^2}{8J^2}\sin^2\frac{JT}{\hbar}$
2. $\frac{\lambda^2}{J^2}\sin^2\frac{JT}{\hbar}$
3. $\frac{\lambda^2}{4J^2}\sin^2\frac{JT}{\hbar}$
4. $\frac{\lambda^2}{16J^2}\sin^2\frac{JT}{\hbar}$

Q.No: 34 CSIR-June-2023

The Hamiltonian of a two-dimensional quantum harmonic oscillator is $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2$ where m and ω are positive constants. The degeneracy of the energy level $\frac{27}{2}\hbar\omega$ is

1. 14
2. 13
3. 8
4. 7

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Q.No: 35 CSIR-JUNE–2023

The radial wavefunction of hydrogen atom with the principal quantum number $n = 2$ and the orbital quantum number $l = 0$ is $R_{20} = N(1 - \frac{r}{2a})e^{-\frac{r}{2a}}$, where N is the normalization constant. The best schematic representation of the probability density $P(r)$ for the electron to be between r and $r + dr$ is



Figure 2: Option 1

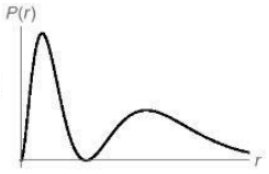


Figure 3: Option 2

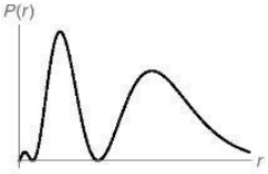


Figure 4: Option 3

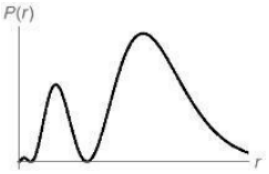


Figure 5: Option 4

Q.No: 36 CSIR-JUNE–2023

Electrons polarized along the x-direction are in a magnetic field $B_1\hat{\mathbf{i}} + B_2(\hat{\mathbf{j}}\cos\ \omega t + \hat{\mathbf{k}}\sin\ \omega t)$, where $B_1 >> B_2$ and ω are positive constants. The value of $\hbar\omega$ for which the polarization-flip process is a resonant one, is

1. $2\mu_B|B_2|$
2. $\mu_B|B_1|$
3. $\mu_B|B_2|$
4. $2\mu_B|B_1|$

Q.No: 37 CSIR-JUNE–2023

The value of $\langle L_x^2\rangle$ in the state $|\phi\rangle$ for which $L^2|\phi\rangle = 6\hbar^2|\phi\rangle$ and $L_z|\phi\rangle = 2\hbar|\phi\rangle$, is

1. 0
2. $4\hbar^2$
3. $2\hbar^2$
4. $\hbar^2$

Q.No: 38 CSIR-JUNE–2024

If $\vec{L}$ is the orbital angular momentum operator and $\vec{\sigma}$ are the Pauli matrices, which of the following operators commutes with $\vec{\sigma}\cdot\vec{L}$?

1. $\vec{L} - \frac{\hbar}{2}\vec{\sigma}$
2. $\vec{L} + \frac{\hbar}{2}\vec{\sigma}$
3. $\vec{L} + \hbar\vec{\sigma}$
4. $\vec{L} - \hbar\vec{\sigma}$

Q.No: 39 CSIR-JUNE–2024

A hydrogen atom is in the state

$$|\psi\rangle = \sqrt{\frac{8}{21}}|\psi_{200}\rangle + \sqrt{\frac{3}{7}}|\psi_{210}\rangle + \sqrt{\frac{4}{21}}|\psi_{311}\rangle,$$

where $|\psi_{nlm}\rangle$ are normalized eigenstates. If $\hat{L}^2$ is measured in this state, the probability of obtaining the value $2\hbar^2$ is:

1. $\frac{13}{21}$
2. $\frac{4}{21}$
3. $\frac{17}{21}$
4. $\frac{3}{7}$

Q.No: 40 CSIR-JUNE–2024

A quantum mechanical system is in the angular momentum state $|l = 4, l_z = 4|$. The uncertainty in L_x is

1. $2\hbar$
2. 0
3. $\hbar\sqrt{2}$
4. $\hbar$

Q.No: 41 CSIR- Dec-2024

An electron is in the spin state

$$|\psi\rangle = \frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix}$$

in the $\hat{S}_z$ basis. A measurement of $\hat{S}_x$ is made on this state. The probabilities of getting $\hbar/2$ and $-\hbar/2$ are

1. $\frac{1}{3}$, $\frac{2}{3}$
2. $\frac{1}{4}$, $\frac{3}{4}$
3. $\frac{1}{2}$, $\frac{1}{2}$
4. $\frac{3}{7}$, $\frac{4}{7}$

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Q.No: 42 CSIR- Dec-2024

Two non–interacting identical spin–$\frac{1}{2}$ particles, each of mass m, are placed in a two–dimensional infinite square well of side L. The single–particle spatial wavefunction is given by

$$\varphi_{n_x,n_y}(x,y) = \frac{2}{L}\sin\Big(\frac{n_x\pi x}{L}\Big)\sin\Big(\frac{n_y\pi y}{L}\Big)$$

where n_x and n_y are positive integers. If the particles are in a total spin state $|j=1, m=0\rangle$, the lowest possible energy eigenvalue is

1. $\frac{5\hbar^2\pi^2}{2mL^2}$
2. $\frac{\hbar^2\pi^2}{mL^2}$
3. $\frac{2\hbar^2\pi^2}{mL^2}$
4. $\frac{7\hbar^2\pi^2}{2mL^2}$

Q.No: 43 CSIR- Dec-2024

A spherical cavity of radius r_0 has an impenetrable wall. A quantum particle of mass m inside the cavity is in its ground state. The pressure exerted on the cavity wall is

1. $\frac{\pi\hbar^2}{4mr_0^5}$
2. $\frac{\pi\hbar^2}{mr_0^5}$
3. $\frac{\pi^2\hbar^2}{2mr_0^5}$
4. $\frac{\pi^2\hbar^2}{4mr_0^5}$

Q.No: 44 CSIR- Dec-2024

For a system of two electrons, define an operator

$$\hat{A} = \frac{3}{a^2}\,(\hat{S}_1\cdot\vec{a})\,(\hat{S}_2\cdot\vec{a}) - \hat{S}_1\cdot\hat{S}_2$$

where $\vec{a}$ is an arbitrary vector, and $\hat{S}_1$ and $\hat{S}_2$ are spin operators. The eigenvalues of $\hat{A}$ (in units of $\hbar^2$) are

1. $-1,\ 1,\ \frac{3}{2},\ \frac{3}{2}$
2. $-1,\ -\frac{1}{2},\ -\frac{1}{2},\ 0$
3. $\frac{1}{2},\ 1,\ \frac{3}{2},\ \frac{3}{2}$
4. $0,\ \frac{1}{2},\ \frac{1}{2},\ -1$

Q.No: 45 CSIR- June-2025

A spin–$\frac{1}{2}$ system is prepared in the initial state

$$|\varphi\rangle = \frac{\sqrt{3}}{2}\,|+\rangle + \frac{1}{2}\,|-\rangle$$

where $|+\rangle$ and $|-\rangle$ are eigenstates of $\hat{S}_z$ with eigenvalues $+\hbar/2$ and $-\hbar/2$ respectively. A measurement of $\hat{S}_z$ is followed by a measurement of $\hat{S}_x$ on the system. What is the probability that the measurement of $\hat{S}_x$ yields a value $+\hbar/2$?

1. $\frac{1}{2}$
2. $\frac{2+\sqrt{3}}{4}$
3. $\frac{2-\sqrt{3}}{4}$
4. $\frac{3}{8}$

Q.No: 46 CSIR- Dec-2025

If $\hat{L}$ is the angular momentum operator for a quantum particle, then $\hat{\vec{L}}\times\hat{\vec{L}}$ is

1. $\hbar^2$
2. $-i\hbar\,\hat{\vec{L}}$
3. 0
4. $i\hbar\,\hat{\vec{L}}$

Q.No: 47 CSIR- Dec-2025

A spin-$\frac{1}{2}$ particle is in a magnetic field $\vec{B} = B_x\hat{x} + B_y\hat{y}$ for which the spin-dependent Hamiltonian is $\hat{H} = -A\,\hat{\vec{S}}\cdot\vec{B}$ (A is a positive constant and $\hat{\vec{S}}$ is the spin-operator). The eigenvalues of the Hamiltonian are

1. $\pm\frac{A\hbar}{2}(B_x + B_y)$
2. $\pm\frac{A\hbar}{2}\sqrt{B_xB_y}$
3. $\pm\frac{A\hbar}{2}\left(B_x^2 + B_y^2\right)^{1/2}$
4. 0

Q.No: 48 CSIR- Dec-2025

A quantum particle of mass m is moving in a potential $V(x,y) = \frac{m\omega^2}{8}\left[5(x^2+y^2) + 8xy\right]$. The lowest energy eigenstate with degeneracy has an energy

1. $\frac{7}{2}\hbar\omega$
2. $\frac{3}{2}\hbar\omega$
3. $4\hbar\omega$
4. $\frac{5}{2}\hbar\omega$

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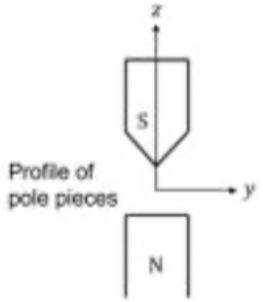
Q.No: 49 CSIR- Dec-2025

For a particle in the angular momentum state $|l = 4, m_l = 2\rangle$, the expectation value of the operator $L_x L_y$ is

1. $-\hbar^2$
2. $\hbar^2$
3. $-i\hbar^2$
4. $i\hbar^2$

Q.No: 50 CSIR- June-2026

A beam of copper atoms (Cu : $[Ar]4s^1 3d^{10}$) is passed through a Stern–Gerlach setup (see figure). If the gradient of the z-component of the magnetic field in the z-direction is 10^3 Tesla/m, then the magnitude of the z-component of the force on an atom is:
(Here μ_B is the Bohr magneton.)



1. $5 \times 10^2 \mu_B$
2. $10^3 \mu_B$
3. 0
4. $2\pi \times 10^3 \mu_B$

Q.No: 51 CSIR- June-2026

Consider the spin matrix S_r projected along the radial direction in spherical polar coordinates (r, θ, ϕ). The probability that a measurement of S_r on the state

$$|\beta\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

will yield the value $\frac{\hbar}{2}$ is:

1. $\sin^2\theta$
2. $\cos^2\theta$
3. $\frac{1}{2}$
4. $\cos^2\frac{\theta}{2}$

Q.No: 52 CSIR- June-2026

The Hamiltonian of a quantum mechanical system is given by $\hat{H} = \alpha\hat{L}^2 + \beta\hbar\hat{L}_z$.Here, α and β are positive constants and $\hat{L}$ represents the orbital angular momentum operator. If l and m are the azimuthal and magnetic quantum numbers then the lowest energy state for this system is:

1. $l = 1, m = -1$ for $\alpha = \beta$
2. $l = 0, m = 0$ for $\alpha < \frac{\beta}{2}$
3. $l = 1, m = -1$ for $\alpha < \frac{\beta}{2}$
4. $l = 1, m = +1$ for $\alpha < \frac{\beta}{2}$

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Question No.	Entrance with year	Answer
1	CSIR-Dec-2014	Option 2
2	CSIR-Dec-2014	Option 2
3	CSIR-June-2015	Option 1
4	CSIR-June-2015	Option 2
5	CSIR-Dec-2015	Option 2
6	CSIR-Dec-2015	Option 1
7	CSIR-Dec-2015	Option 4
8	CSIR-June-2016	Option 1
9	CSIR-June-2016	Option 1
10	CSIR-June-2017	Option 2
11	CSIR-Dec-2017	Option 4
12	CSIR-June-2018	Option 3
13	CSIR-June-2018	Option 4
14	CSIR-June-2018	Option 3
15	CSIR-Dec-2018	Option 2
16	CSIR-Dec-2018	Option 4
17	CSIR-Dec-2018	Option 1
18	CSIR-Dec-2018	Option 2
19	CSIR-Dec-2018	Option 4
20	CSIR-June-2019	Option 1
21	CSIR-Dec-2019	Option 3
22	CSIR-Dec-2019	Option 4
23	CSIR-Dec-2019	Option 1
24	CSIR-June-2020	Option 2
25	CSIR-June-2020	Option 3
26	CSIR-Feb-2022	Option 3
27	CSIR-Sep-2022	Option 1
28	CSIR-Sep-2022	Option 4
29	CSIR-Sep-2022	Option 4
30	CSIR-Dec-2023	Option 2
31	CSIR-Dec-2023	Option 1
32	CSIR-Dec-2023	Option 3
33	CSIR-Dec-2023	Option 2
34	CSIR-June-2023	Option 4
35	CSIR-June-2023	Option 1
36	CSIR-June-2023	Option 4
37	CSIR-June-2023	Option 4
38	CSIR-June-2024	Option 2
39	CSIR-June-2024	Option 1
40	CSIR-June-2024	Option 3
41	CSIR-Dec-2024	Option 3
42	CSIR-Dec-2024	Option 4
43	CSIR-Dec-2024	Option 1
44	CSIR-Dec-2024	Option 4
45	CSIR-June-2025	Option 1
46	CSIR-Dec-2025	Option 4
47	CSIR-Dec-2025	Option 3
48	CSIR-Dec-2025	Option 4
49	CSIR-Dec-2025	Option 4
50	CSIR-June-2026	Option 2
51	CSIR-June-2026	Option 1
52	CSIR-June-2026	Option 3

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