3D and Angular Momentum JEST and TIFR

https://snboseplc.com/3d-and-angular-momentum-jest-tifr/

JEST Questions

Q.No: 0 JEST-2012

Consider a spin-1/2 particle in the presence of a homogeneous magnetic field of magnitude B along z-axis which is prepared initially in a state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ at time $t = 0$. At what time t will the particle be in the state $-|\Psi\rangle$ (μ_B is Bohr magneton)?

1. $t = \frac{\pi\hbar}{\mu_B B}$
2. $t = \frac{2\pi\hbar}{\mu_B B}$
3. $t = \frac{\pi\hbar}{2\mu_B B}$
4. Never

Q.No:1 JEST-2012

Consider a particle of mass m moving inside a two-dimensional square box whose sides are described by the equations $x = 0, x = L, y = 0, y = L$. What is the lowest eigenvalue of an eigenstate which changes sign under the exchange of x and y?

1. $\hbar^2/(mL^2)$
2. $3\hbar^2/(2mL^2)$
3. $5\hbar^2/(2mL^2)$
4. $7\hbar^2/(2mL^2)$

Q.No:2 JEST-2012

Define $\sigma_x = (f^\dagger + f)$, and $\sigma_y = -i(f^\dagger - f)$, where the σ's are Pauli spin matrices and $f, f^\dagger$ obey anticommutation relations $\{f, f\} = 0, \{f, f^\dagger\} = 1$. Then σ_z is given by

1. $f^\dagger f - 1$
2. $2f^\dagger f - 1$
3. $2f^\dagger f + 1$
4. $f^\dagger f$

Q.No:3 JEST-2013

If J_x, J_y, J_z are angular momentum operators, the eigenvalues of the operator $(J_x + J_y)/\hbar$ are

1. real and discrete with rational spacing
2. real and discrete with irrational spacing
3. real and continuous
4. not all real

Q.No:4 JEST-2013

Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with $m = +1, 0, -1$. If L_z^2 is measured in this state yielding a result 1, what is the state after the measurement?

1. $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
2. $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$
3. $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
4. $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

Q.No:5 JEST-2013

What are the eigenvalues of the operator $H = \vec{\sigma} \cdot \vec{a}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{a}$ is a vector?

1. $a_x + a_y$ and a_z
2. $a_x + a_z \pm i a_y$
3. $\pm(a_x + a_y + a_z)$
4. $\pm|\vec{a}|$

Q.No:6 JEST-2014

Suppose a spin 1/2 particle is in the state

$$|\psi\rangle = \frac{1}{\sqrt{6}} \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$$

If S_x (x component of the spin angular momentum operator) is measured what is the probability of getting $+\hbar/2$?

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1. 1/3
2. 2/3
3. 5/6
4. 1/6

Q.No:7 JEST-2014

Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|l, m\rangle$. Let $A = \hat{n} \cdot \vec{L}$ denote an operator, where $\hat{n}$ is a unit vector parametrized in terms of two angles as $(n_x, n_y, n_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The width ΔA in $|l, m\rangle$ state is:

1. $\sqrt{\frac{l(l+1)-m^2}{2}}\hbar\cos\theta$
2. $\sqrt{\frac{l(l+1)-m^2}{2}}\hbar\sin\theta$
3. $\sqrt{l(l+1)-m^2}\hbar\sin\theta$
4. $\sqrt{l(l+1)-m^2}\hbar\cos\theta$

Q.No:8 JEST-2016

The wavefunction of a hydrogen atom is given by the following superposition of energy eigenfunctions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers):

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

1. $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$
2. $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$
3. $\frac{101}{504}$ and $\hbar^2$
4. $\frac{229}{504}$ and $\hbar^2$

Q.No:9 JEST-2016

A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $(1/\sqrt{10})\{2, \sqrt{2}, 2i\}$ in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_x|\psi\rangle$?

1. 1/2
2. 1/3
3. 1/4
4. 1/6

Q.No:10 JEST-2016

A spin-1/2 particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $(|1\rangle + |2\rangle)/\sqrt{2}$ at time $t = 0$. It evolves to the state described by the ket $(|1\rangle - |2\rangle)/\sqrt{2}$ in time T. The minimum energy difference between two levels is:

1. $h/6T$
2. $h/4T$
3. $h/2T$
4. h/T

Q.No:11 JEST-2016

In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is:

1. 1/2
2. 1
3. 2
4. 3/2

Q.No:12 JEST-2016

A spin 1/2 particle is in a state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z operator. The expectation value of the spin angular momentum measured along x direction is:

1. $\hbar$
2. $-\hbar$
3. 0
4. $\hbar/2$

Q.No:13 JEST-2016

If the direction with respect to a right-handed cartesian coordinate system of the ket vector $|z, +\rangle$ is $(0, 0, 1)$, then the direction of the ket vector obtained by application of rotations: $\exp(-i\sigma_z\pi/2)\exp(i\sigma_y\pi/4)$, on the ket $|z, +\rangle$ is (σ_y, σ_z are the Pauli matrices):

1. $(0, 1, 0)$
2. $(1, 0, 0)$
3. $(1, 1, 0)/\sqrt{2}$
4. $(1, 1, 1)/\sqrt{3}$

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Q.No:14 JEST-2017

If $\rho = [I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z)]/2$, where σ's are the Pauli matrices and I is the identity matrix, then the trace of ρ^{2017} is

1. 2^{2017}.
2. 2^{-2017}.
3. 1.
4. $\frac{1}{2}$.

Q.No:15 JEST-2018

A quantum particle of mass m is moving on a horizontal circular path of radius a. The particle is prepared in a quantum state described by the wavefunction

$$\psi = \sqrt{\frac{4}{3\pi}} \cos^2 \phi,$$

ϕ being the azimuthal angle. If a measurement of the z-component of orbital angular momentum of the particle is carried out, the possible outcomes and the corresponding probabilities are

1. $L_z = 0, \pm\hbar, \pm 2\hbar$ with $P(0) = 1/5, P(\pm\hbar) = 1/5$ and $P(\pm 2\hbar) = 1/5$
2. $L_z = 0$ with $P(0) = 1$
3. $L_z = 0, \pm\hbar$ with $P(0) = 1/3$ and $P(\pm\hbar) = 1/3$
4. $L_z = 0, \pm 2\hbar$ with $P(0) = 2/3$ and $P(\pm 2\hbar) = 1/6$

Q.No:16 JEST-2019

For a spin-$\frac{1}{2}$ particle placed in a magnetic field B, the Hamiltonian is $\mathcal{H} = -\gamma B S_y = -\omega S_y$, where S_y is the y-component of the spin operator. The state of the system at time $t = 0$ is $|\psi(t = 0)\rangle = |+\rangle$, where $S_z|\pm\rangle = \pm\frac{\hbar}{2}|\pm\rangle$. At a later time t, if S_z is measured then what is the probability to get a value $-\frac{\hbar}{2}$?

1. $\cos^2(\omega t)$
2. $\sin^2(\omega t)$
3. 0
4. $\sin^2(\omega t/2)$

Q.No:17 JEST-2020

A particle in a spherically symmetric potential is known to be in an eigenstate of $\vec{L}^2$ and L_z with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$, respectively. What is the value of $\langle l, m|L_x^2|l, m\rangle$?

1. $\frac{\hbar^2}{2}(l^2 + l + m^2)$
2. $\frac{\hbar^2}{3}(l^2 + l)$
3. $\hbar^2(l^2 + l - m^2)$
4. $\frac{\hbar^2}{2}(l^2 + l - m^2)$

Q.No:18 JEST-2020

The wavefunction of a particle subjected to a spherically symmetric potential $V(r)$ is given by $\psi(\vec{r}) = (x - y + 2z)f(r)$. Which one of the following statements is true about $\psi(\vec{r})$?

1. It is an eigenfunction of $\vec{L}^2$ with $l = 0$
2. It is an eigenfunction of $\vec{L}^2$ with $l = 1$
3. It is an eigenfunction of $\vec{L}^2$ with $l = 2$
4. It is not an eigenfunction of $\vec{L}^2$

Q.No:19 JEST-2020

Consider a quantum particle of mass m moving in a potential

$$V(x, y) = \begin{cases} \frac{1}{2}m\omega^2(x^2 + y^2), & \text{for } x > 0, y > 0 \\ \infty, & \text{otherwise.} \end{cases}$$

What is the degeneracy of the energy state $9\hbar\omega$, where $\omega > 0$ measures the strength of the potential?

1. 4
2. 2
3. 10
4. 5

Q.No:20 JEST-2021

If $\vec{\mathbf{L}}$ is the angular momentum operator in quantum mechanics, the value of $\vec{\mathbf{L}} \times \vec{\mathbf{L}}$ will be:

1. 0
2. $i\hbar\vec{\mathbf{L}}$
3. $|\vec{\mathbf{L}}|^2$
4. $\hbar\vec{\mathbf{L}}$

Q.No:21 JEST-2016

If $Y_{xy} = \frac{1}{\sqrt{2}}(Y_{2,2} - Y_{2,-2})$ where $Y_{l,m}$ are spherical harmonics, then which of the following is true?

1. Y_{xy} is an eigenfunction of both L^2 and L_z
2. Y_{xy} is an eigenfunction of L^2 but not L_z
3. Y_{xy} is an eigenfunction of L_z but not L^2
4. Y_{xy} is not an eigenfunction of either L^2 or L_z

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Q.No:22 JEST-2022

The wavefunction of the electron in a Hydrogen atom in a particular state is given by $\pi^{-1/2}a_0^{-3/2}\exp(-r/a_0)$. Which of the following figures qualitatively depicts the probability ($P(r)$) of the electron to be within a distance r from the nucleus?

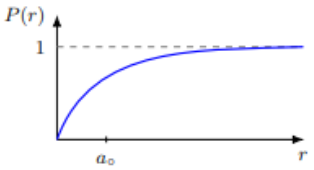


Figure 1: (a)

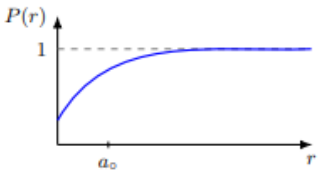


Figure 2: (b)

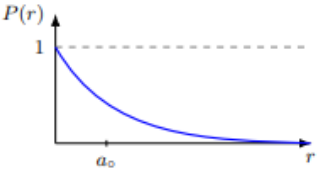


Figure 3: (c)

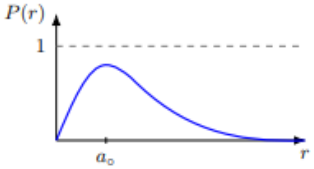


Figure 4: (d)

Q.No:23 JEST-2022

Let $M = 2\mathbb{I} + \sigma_x + i\sigma_y + \sigma_z$ is a 2×2 square matrix, where, σ_α denotes α^{th} Pauli matrix, and $\mathbb{I}$ denotes the 2×2 identity matrix. It is given that $|u\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|v\rangle = \begin{pmatrix}1\\-1\end{pmatrix}$ are column vectors. What is the value of $\langle u|\sqrt{M}|v\rangle$?

Q.No:24 JEST-2023

Consider a spin-1/2 particle in the quantum state $|\psi(\beta,\alpha)\rangle = cos\ (\frac{\beta}{2})\ |\uparrow\rangle + sin\ (\frac{\beta}{2})\ e^{i\alpha}\ |\downarrow\rangle$ where $0\le\beta\le\pi$ and $0\le\alpha\le 2\pi$. For which values of (δ,γ) is the state $|\psi(\delta,\gamma)\rangle$ orthogonal to $|\psi(\beta,\alpha)\rangle$?

1. $(\pi+\beta,\pi-\alpha)$
2. $(\pi-\beta,\pi-\alpha)$
3. $(\pi+\beta,\pi+\alpha)$
4. $(\pi-\beta,\pi+\alpha)$

Q.No : 25 JEST-2023

Consider a spin-1 system whose $\hat{S}_z$ eigenstates are given by $|-1\rangle,|0\rangle,|+1\rangle$ corresponding to the eigenvalues $-\hbar,0,\hbar$. The normalized general state $|\psi\rangle$ of the system can be expressed as

$$|\psi\rangle = c_{-1}|-1\rangle + c_0|0\rangle + c_{+1}|+1\rangle$$

and c_{-1},c_0,c_{+1} are complex numbers. Subjected to the condition $\langle\psi|\hat{S}_z|\psi\rangle = 0$ which of the following statements is true?

1. $|c_{+1}|^2+2|c_0|^2=1$
2. $|c_{-1}|^2+2|c_0|^2=1$
3. $2|c_{-1}|^2+|c_{+1}|^2=1$
4. $2|c_{-1}|^2+|c_0|^2=1$

Q.No : 26 JEST-2023

Consider the operator $S\cdot\hat{n}$ with eigenkets $|\pm\rangle_{\hat{n}}$ and eigenvalues $\pm\frac{\hbar}{2}$ where $\hat{n}$ is a unit vector and S is the spin operator. A partially polarized beam of spin-$\frac{1}{2}$ particles contains a 25-75 mixture of two pure ensembles, one with $|+\rangle_{\hat{z}}$ and the other with $|+\rangle_{\hat{x}}$ respectively. What is the ensemble average of

$$\frac{S\cdot\hat{x}}{\hbar}?$$

1. $\frac{1}{3}$
2. $\frac{3}{8}$
3. $\frac{1}{4}$
4. $\frac{3}{16}$

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Q.No : 27 JEST-2024

Consider a particle of mass m moving in a three-dimensional delta-function potential well $V(\vec{r}) = -\alpha\delta^3(\vec{r})$, where $\alpha > 0$. Which of the following is an allowed expression for the energy of a bound state for some dimensionless proportionality constant $\beta > 0$?

1. $-\frac{\beta\hbar^6}{\alpha^2 m^3}$
2. $\frac{\beta\hbar^6}{\alpha^2 m^3}$
3. $-\frac{\beta\alpha^2 m}{\hbar^2}$
4. $\frac{\beta\alpha^2 m}{\hbar^2}$

Q.No : 28 JEST-2024

A two-level quantum system has the Hamiltonian

$$H = \hbar\omega_0 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

. At $t = 0$, the system is in the state

$$|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

. What is the earliest time $t > 0$ at which a measurement of

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

will yield the value -1 with probability one?

1. Never
2. $\frac{2\pi}{\omega_0}$
3. $\frac{\pi}{\omega_0}$
4. $\frac{\pi}{2\omega_0}$

Q.No : 29 JEST-2024

Two electrons have orbital angular momentum quantum numbers $l_1 = 3$ and $l_2 = 2$, respectively. Let $L^z = L_1^z + L_2^z$, where L_1^z and L_2^z are the z-components of the respective angular momentum operators. How many linearly independent states have L^z quantum number $m = 2$?

1. 3
2. 4
3. 11
4. 0

Q.No : 30 JEST-2025

Three observers successively measure the spin of a given proton along the z–axis, x–axis and again z–axis, respectively. The first observer finds the spin projection to be $+\frac{1}{2}$. Assuming no other factors, what is the probability that the third observer finds the spin projection to be $-\frac{1}{2}$?

1. 0
2. 1
3. 0.5
4. None of the others

Q.No : 31 JEST-2025

Given the mass of the proton $m_p \simeq 1836\, m_e$ and mass of the deuteron $m_d \simeq 3670\, m_e$, where m_e is the electron mass, find the fractional shift (in parts per million, to the nearest integer) of the ground state energy of the deuterium atom as compared to H-atom.

Q.No : 32 JEST-2026

What is

$$\left\langle \frac{1}{r} \right\rangle$$

in the state $|n, l, m\rangle$ of a hydrogen atom in terms of the Bohr radius a_0?

(A) $\frac{1}{n^2 a_0}$

(B) $\frac{1}{2n^2 a_0}$

(C) $\frac{1}{4n^2 a_0}$

(D) $\frac{1}{2na_0}$

Q.No : 33 JEST-2026

An operator A is expressed in terms of angular momentum operators as,

$$A = L^2 + aL_z^2 + b(L_+L_- + L_-L_+),$$

where $L_\pm = L_x \pm iL_y$, and a and b are arbitrary constants. Under what condition does A commute with L_x^2?

(A) $a = b$

(B) $a = 2b$

(C) $2a = b$

(D) $a = -b$

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TIFR Questions

Q.No: 1 TIFR-2012

The normalized wavefunctions of a Hydrogen atom are denoted by $\psi_{n,\ell,m}(\vec{x})$, where n, ℓ and m are, respectively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\Psi(\vec{x}) = \frac{1}{3}\psi_{1,0,0}(\vec{x}) + \frac{2}{3}\psi_{2,1,0}(\vec{x}) + \frac{2}{3}\psi_{3,2,-2}(\vec{x})$$

The expectation value $\langle E \rangle$ of the energy of this electron, in electron-Volts eV) will be approximately

1. -1.5
2. -3.7
3. -13.6
4. -80.1
5. $+13.6$

Q.No:2 TIFR-2012

The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

1. free particles
2. particles in a box
3. harmonic oscillators
4. rigid rotators
5. hydrogenic atoms

Q.No: 3 TIFR-2013

An energy eigenstate of the Hydrogen atom has the wave function

$$\psi_{n\ell m}(r,\theta,\varphi) = \frac{1}{81\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \sin\theta\cos\theta\exp\left[-\left(\frac{r}{3a_0} + i\varphi\right)\right]$$

where a_0 is the Bohr radius. The principal (n), azimuthal (ℓ) and magnetic (m) quantum numbers corresponding to this wave function are

1. $n = 3, \ell = 2, m = 1$
2. $n = 2, \ell = 1, m = 1$
3. $n = 3, \ell = 2, m = -1$
4. $n = 2, \ell = 1, m = \pm 1$

Q.No: 4 TIFR-2014

Consider the Hamiltonian

$$H = f\vec{\sigma}\cdot\vec{x}$$

Here $\vec{x}$ is the position vector, f is a constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices. The energy eigenvalues are

1. $f(\sqrt{x^2+y^2} \pm z)$
2. $f(x \pm iy)$
3. $\pm f\sqrt{x^2+y^2+z^2}$
4. $\pm f(x+y+z)$

Q.No: 5 TIFR-2014

A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\varphi$$

where θ and φ are the usual polar angles. If two successive measurements of L_z are made on this rotator, the probability that the second measurement will yield the value $+\hbar$ is

1. 0.25
2. 0.33
3. 0.5
4. negligible

Q.No: 6 TIFR-2014

A particle in the $2s$ state of hydrogen has the wave function

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2}\pi}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

where r is the radial coordinate w.r.t. the nucleus as origin and a_0 is the Bohr radius. The probability P of finding the electron somewhere inside a sphere of radius λa_0 centred at the nucleus, is best described by the graph

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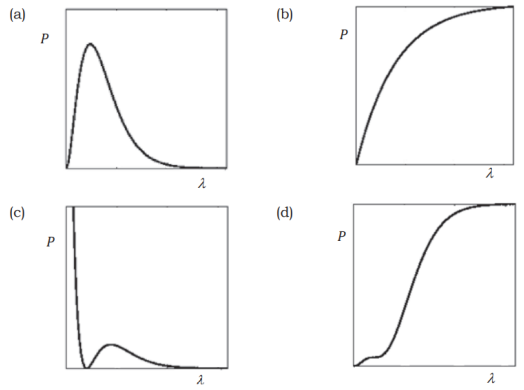


Figure 5: Graph for Q.No: 6 TIFR-2014

Q.No: 7 TIFR-2014

In a Stern-Gerlach experiment with spin-1/2 particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another Stern-Gerlach apparatus but with its magnets rotated by 90 degrees counterclockwise about the axis of the beam direction. Which of the following shows what happens on the second screen?

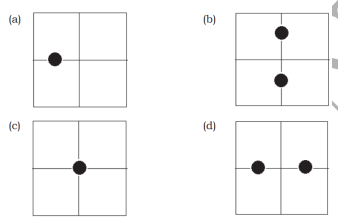


Figure 6: Figure for Q.No: 7 TIFR-2014

Q.No : 8 TIFR-2015

The ground state energy of a particle of mass m in a three-dimensional cubical box of side ℓ is not zero but $3h^2/8m\ell^2$. This is because

1. the ground state has no nodes in the interior of the box.
2. this is the most convenient choice of the zero level of potential energy.
3. position and momentum cannot be exactly determined simultaneously.
4. the potential at the boundaries is not really infinite, but just very large.

Q.No: 9 TIFR-2015

A one-dimensional box contains a particle whose ground state energy is ϵ. It is observed that a small disturbance causes the particle to emit a photon of energy $h\nu = 8\epsilon$, after which it is stable. Just before emission, a possible state of the particle in terms of the energy eigenstates $\{\psi_1, \psi_2, ...\}$ would be

1. $\frac{\psi_1 - \psi_2}{\sqrt{2}}$
2. $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$
3. $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$
4. $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$

Q.No: 10 TIFR-2015

An rigid rotator has the wave function

$$\psi(\theta, \varphi) = N[2iY_{1,0}(\theta, \varphi) + (2+i)Y_{2,-1}(\theta, \varphi) + 3iY_{1,1}(\theta, \varphi)]$$

where $Y_{l,m}(\theta, \varphi)$ are the spherical harmonics, and N is a normalization constant. If $\vec{L}$ is the orbital angular momentum operator, and $L_\pm = L_x \pm iL_y$ the expectation value of L_+L_- is

1. $21\hbar^2/9$
2. $23\hbar^2/9$
3. $25\hbar^2/9$
4. 0

Q.No: 11 TIFR-2016

A quantum mechanical plane rotator consists of two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the x-y plane about their centre of mass. Suppose that the initial state of the rotor is given by

$$\psi(\varphi, t = 0) = A\cos^2\varphi,$$

where φ is the angle between one mass and the x axis, while A is a normalization constant. Find the expectation value of $3\hat{L}_Z^2$ in this state, in units of $\hbar^2$.

Q.No: 12 TIFR-2017

Electrons in a given system of hydrogen atoms are described by the wave function

$$\psi(r, \theta, \varphi) = 0.8\Psi_{100} + 0.6e^{i\pi/3}\Psi_{311}$$

where the $\Psi_{n\ell m}$ denote normalized energy eigenstates. If $(\hat{L}_x, \hat{L}_y, \hat{L}_z)$ are the components of the orbital angular momentum operator, the expectation value of $\hat{L}_x^2$ in this system is

1. $1.5\hbar^2$
2. $0.36\hbar^2$
3. $0.18\hbar^2$
4. Zero

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Q.No: 13 TIFR-2018

A particle is in the ground state of a cubical box of side ℓ. Suddenly one side of the box changes from ℓ to 4ℓ. If p is the probability of finding the particle in the ground state of the new box, what is $1000p$?

Q.No: 14 TIFR-2018

An electron is in the $2s$ level of the hydrogen atom, with the radial wave-function

$$\psi(r) = \frac{1}{2\sqrt{2}a_0^{3/2}}\left(2 - \frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right).$$

The probability $P(r)$ of finding this electron between distances r to $r + dr$ from the centre is best represented by the sketch

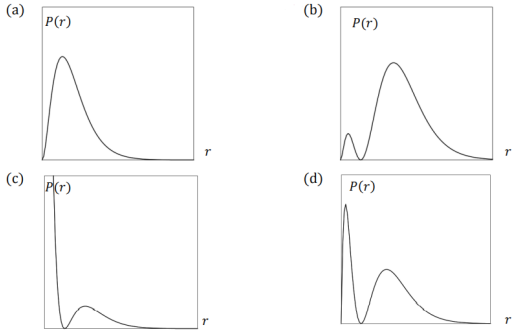


Figure 7: Graph for Q.No: 14 TIFR-2018

Q.No: 15 TIFR-2019

An electron in a hydrogen atom is in a state described by the wavefunction:

$$\Psi(\vec{r}) = \frac{1}{\sqrt{10}}\psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}}\psi_{21,-1}(\vec{x})$$

where $\psi_{n\ell m}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n, angular quantum number ℓ and magnetic quantum number m.

Neglecting the spin-orbit interaction, the expectation values $\hat{L}_Z$ and $\hat{L}^2$ for this state are

1. $3\hbar/10, 9\hbar^2/5$
2. $3\hbar/5, 9\hbar^2/10$
3. $3\hbar/4, 9\hbar^2/25$
4. $8\hbar/10, 3\hbar^2/5$

Q.No: 16 TIFR-2020

The wave function of a particle subjected to a three-dimensional spherically-symmetric potential $V(r)$ is given by

$$\psi(\vec{x}) = (x + y + 3z)f(r)$$

The expectation value for the operator $\vec{L}^2$ for this state is

1. $\hbar^2$
2. $2\hbar^2$
3. $5\hbar^2$
4. $11\hbar^2$

Q.No: 17 TIFR-2020

A fermion of mass m, moving in two dimensions, is strictly confined inside a square box of side ℓ. The potential inside is zero. A measurement of the energy of the fermion yields the result

$$E = \frac{65\pi^2\hbar^2}{2m\ell^2}$$

The degeneracy of this energy state is

1. 2
2. 4
3. 8
4. 16

Q.No: 18 TIFR-2012

If we model the electron as a uniform sphere of radius r_e, spinning uniformly about an axis passing through its centre with angular momentum $L_e = \hbar/2$, and demand that the velocity of rotation at the equator cannot exceed the velocity c of light in vacuum, then the minimum value of r_e is

1. $19.2fm$
2. $0.192fm$
3. $4.8fm$
4. $1960fm$
5. $480fm$

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Q.No: 19 TIFR-2021

An electron is confined to a two-dimensional square box with the following potential

$$V = \begin{cases} 0 & \text{for } 0 < x < L \text{ and } 0 < y < L, \\ \infty & \text{otherwise} \end{cases}$$

The probability distribution of the electron in one of its eigenstates is shown below

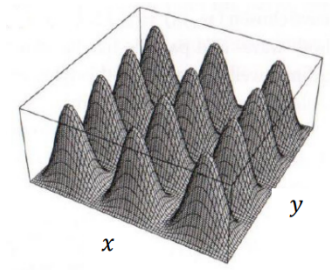


Figure 8: Probability distribution for Q.No: 19 TIFR-2021

How many total different eigenstates of the electron have the same energy as this state?

1. 4
2. 2
3. 6
4. 1

Q.No: 20 TIFR-2021

The Hamiltonian of a spin-1/2 particle in a magnetic field $\vec{B}$ is given by $H = -\mu\vec{S}\cdot\vec{B}$, where the components of the spin operator $\vec{S}$ have eigenvalues $\pm\hbar/2$. The spin is pointing in the $+\hat{x}$ direction, when a magnetic field $\vec{B} = B\hat{y}$ is turned on. After a time $t = \pi/2\mu B$, the spin will be pointing along the direction.

1. $+\hat{z}$
2. $-\hat{z}$
3. $-\hat{x}$
4. $\hat{x} + \hat{z}$

Q.No : 21 TIFR-2021

An electron moves in a hydrogen atom potential in a state $|\Psi\rangle$ that has the wave function

$$\Psi(r,\theta,\varphi) = NR_{21}(r)[2iY_1^{-1}(\theta,\varphi)+(2+i)Y_1^0(\theta,\varphi)+3iY_1^1(\theta,\varphi)]$$

where N is a normalization constant, $R_{nl}(r)$ is the radial wave function and the $Y_l^m(\theta,\varphi)$ are spherical harmonics.

The expectation value of $\hat{L}_z$, i.e. the $\hat{z}$-component of the angular momentum operator is

1. $\frac{5}{18}\hbar$
2. $\frac{4}{18}\hbar$
3. $\frac{9}{18}\hbar$
4. $\frac{13}{18}\hbar$

Q.No: 22 TIFR-2022

The Principle of Linear Superposition of electron states in quantum mechanics is nicely illustrated by the

1. Davisson-Germer experiment
2. Compton scattering experiment
3. Franck-Hertz experiment
4. Millikan oil-drop experiment

Q.No: 23 TIFR-2022

In a matrix mechanics formulation, a spin-1 particle has angular momentum components

$$L_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & -1 \\ 1 & \sqrt{2} & 0 \\ -1 & 0 & -\sqrt{2} \end{pmatrix} \quad L_z = \frac{\hbar}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

It follows that $L_y =$

1. $\frac{\hbar}{2}\begin{pmatrix} 0 & -i & i \\ i & 0 & -i\sqrt{2} \\ -i & i\sqrt{2} & 0 \end{pmatrix}$
2. $\frac{\hbar}{2}\begin{pmatrix} 0 & i & -i \\ -i & 0 & i\sqrt{2} \\ i & -i\sqrt{2} & 0 \end{pmatrix}$
3. $\sqrt{2}\hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
4. $\sqrt{2}\hbar\begin{pmatrix} 0 & -\sqrt{2} & 0 \\ -\sqrt{2} & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Q.No: 24 TIFR-2023

Consider an electron with mass m_e, charge $-e$ and spin 1/2, whose spin angular momentum operator is given by $\hat{\vec{S}} = \frac{\hbar}{2}\vec{\sigma}$ This electron is placed in a magnetic field $\vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, where all three components (B_x, B_y, B_z) are nonvanishing.

At time $t = 0$, the electron is at rest in the $S_z = \hbar/2$ state. The earliest time when the state of the spin will be orthogonal to the initial state is

1. $\frac{2m_e}{ge\vert\vec{B}\vert}$

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2. infinity, i.e., it will never be orthogonal.
3. $\frac{4m_e}{ge|\vec{B}|}$
4. dependent on the direction of the magnetic field $\vec{B}$

Q.No: 25 TIFR-2024

A particle of mass m is subjected to a force $\vec{F}(r)$ such that the wavefunction $\phi(\vec{p},t)$ satisfies the momentum-space Schrödinger equation

$$\left(\frac{\vec{p}^2}{2m} - a\vec{\nabla}_p^2\right)\phi(\vec{p},t) = i\hbar\frac{\partial\phi(\vec{p},t)}{\partial t}$$

where a is a real constant and

$$\vec{\nabla}_p^2 = \frac{\partial^2}{\partial p_x^2} + \frac{\partial^2}{\partial p_y^2} + \frac{\partial^2}{\partial p_z^2}$$

It follows that $\vec{F}(r)$ equals

1. $-\frac{2a}{\hbar^2}\vec{r}$
2. $\frac{2a}{\hbar^2}\vec{r}$
3. $-\frac{a\hbar^2}{r^3}\vec{r}$
4. $\frac{a\hbar^2}{r^3}\vec{r}$

Q.No: 26 TIFR-2024

An electron confined in a two-dimensional square box, is in the ground state. The length of the side of this square is unknown, but it is seen that the electron jumps to the first excited energy state by absorbing electromagnetic radiation of wavelength 4,040 nm. What is the length of one side of the square well?

1. 1.91 nm
2. 1.68 nm
3. 2.55 nm
4. 3.82 nm

Q.No: 27 TIFR-2024

A quantum-mechanical state of a particle, with Cartesian coordinates x, y, and z, is described by the normalized wave function

$$\psi(x,y,z) = \frac{\alpha^{5/2}}{\sqrt{\pi}} z e^{-\alpha\sqrt{(x^2+y^2+z^2)}}$$

For this state what are the angular quantum number ℓ, L^2 and L_z respectively?

1. $0; 0; 0$
2. $1; 2\hbar^2; \hbar$
3. $1; 2\hbar^2; 0$
4. $2; 6\hbar^2; 0$

Q.No: 28 TIFR-2025

Consider a stationary electron in a uniform, time–independent magnetic field of strength $B_0/4$ oriented in the $\hat{z}$–direction. The Hamiltonian for this system is expressed as

$$H = -\frac{e}{m}\vec{S}\cdot\vec{B}$$

where $\vec{S}$ is the spin–$\frac{1}{2}$ operator for electrons. The initial electron spin is oriented in the $\hat{x}$–direction. The spin precession frequency of the electrons is:

1. $\frac{|e|B_0}{4m}$
2. $\frac{|e|B_0}{8m}$
3. $\frac{|e|B_0}{2m}$
4. 0

Q.No: 29 TIFR-2025

Let $|nlm\rangle$ denote the energy eigenstates of nonrelativistic hydrogen atoms without spin, and a_0 is the Bohr radius. The matrix element

$$\langle n=2,\, l=1,\, m_z=0 \mid \hat{x} \mid n=2,\, l=0,\, m_z=0\rangle$$

is:

1. 0
2. $\sqrt{2}\,a_0$
3. a_0
4. $\sqrt{3}\,a_0$

Q.No: 30 TIFR-2025

Consider a free particle in 3 spatial dimensions described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m}$$

It is initially in a state described by a normalized wavefunction

$$\psi(\mathbf{r}, t=0) = \left(\frac{\gamma}{\pi}\right)^{3/4} e^{-\gamma r^2/2}$$

What is the probability density of finding the particle with energy E at time t?

(Hint: Express the wavefunction in momentum space.)

(The following integral might be useful:

$$\int_{-\infty}^{+\infty} dx\, \frac{1}{\sqrt{2\pi}} e^{-ikx} e^{-\gamma x^2/2} = \frac{1}{\sqrt{\gamma}} e^{-k^2/(2\gamma)}$$

1. $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{2mE}\, e^{-2mE/(\gamma\hbar^2)}$

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2. $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{\frac{2m\hbar}{t}}\, e^{-2mE/(\gamma\hbar^2)}$

3. $\frac{2m}{\hbar}\frac{1}{\sqrt{2mE}}(\gamma\pi)^{-1/2}e^{-2mE/(\gamma\hbar^2)}$

4. $\frac{2\pi m}{\hbar^2}(\gamma\pi)^{-1}e^{-2mE/(\gamma\hbar^2)}$

Q.No: 31 TIFR-2026

Consider a 2×2 matrix defined as:

$$A = [a_0 + i\,\vec{\sigma}\cdot\vec{a}][a_0 - i\,\vec{\sigma}\cdot\vec{a}]^{-1}$$

where a_0 is a non-zero real number, $\vec{a}$ is a three-dimensional non-zero real vector, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices. The matrix A:

(a) Is Unitary but not Hermitian

(b) Is Hermitian but not Unitary

(c) Is Unitary and Hermitian

(d) Is neither Unitary nor Hermitian

Q.No: 32 TIFR-2026

Hydrogen atom has a decay rate from $n = 2$ state to $n = 1$ state as about 6×10^8 decays per second. If the energy level expression for the hydrogen atom is $E_n = \frac{13.6}{n^2}$ eV the uncertainty in the wavelength of this emission would be closest to

(a) 30 fm

(b) 0.1 fm

(c) 122 nm

(d) 19.4 nm

Q.No: 33 TIFR-2026

A quantum particle of mass m in three-dimensions is governed by the Hamiltonian:

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + \frac{1}{2}m\omega^2(x^2+y^2+z^2) + \lambda(x^6+y^6)$$

with $\lambda > 0$. If $\psi_3(\vec{r})$ is the third excited state of the system, which of the following is correct?

(a) $|\psi_3(\vec{r})|^2 = |\psi_3(-\vec{r})|^2$

(b) $\psi_3(\vec{r})$ is an eigenstate of L^2

(c) $\psi_3(\vec{r})$ is an eigenstate of L_z

(d) $\psi_3(\vec{r})$ is an eigenstate of p_z

Q.No: 34 TIFR-2026

In the standard $|S^2, S_z\rangle$ eigenbasis of a spin 1 particle, the matrix corresponding to the operator $\hat{S}_x^2 - \hat{S}_y^2$ is given by:

(a) $\hbar^2\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}$

(b) $\hbar^2\begin{pmatrix}1&0&0\\0&2&0\\0&0&1\end{pmatrix}$

(c) $\hbar^2\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$

(d) $\hbar^2\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}$

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Question No.	Entrance with year
Q.No: 0 JEST-2012	Option b
Q.No: 1 JEST-2012	Option c
Q.No: 2 JEST-2012	Option b
Q.No: 3 JEST-2013	Option b
Q.No: 4 JEST-2013	Option b
Q.No: 5 JEST-2013	Option d
Q.No: 6 JEST-2014	Option c
Q.No: 7 JEST-2014	Option b
Q.No: 8 JEST-2016	Option a
Q.No: 9 JEST-2016	Option d
Q.No: 10 JEST-2016	Option c
Q.No: 11 JEST-2016	Option b
Q.No: 12 JEST-2016	Option d
Q.No: 13 JEST-2016	Option b
Q.No: 14 JEST-2017	Option c
Q.No: 15 JEST-2018	Option d
Q.No: 16 JEST-2019	Option d
Q.No: 17 JEST-2020	Option d
Q.No: 18 JEST-2020	Option b
Q.No: 19 JEST-2020	Option a
Q.No: 20 JEST-2021	Option b
Q.No: 21 JEST-2016	Option b
Q.No: 22 JEST-2022	Option a
Q.No: 23 JEST-2022	1
Q.No: 24 JEST-2023	Option d
Q.No: 25 JEST-2023	Option d
Q.No: 26 JEST-2023	Option b
Q.No: 27 JEST-2024	Option a
Q.No: 28 JEST-2024	Option d
Q.No: 29 JEST-2024	Option b
Q.No: 30 JEST-2025	Option C
Q.No: 31 JEST-2025	272
Q.No: 32 JEST-2026	Option A
Q.No: 33 JEST-2026	Option B

Question No.	Entrance with year
Q.No: 1 TIFR-2012	Option b
Q.No: 2 TIFR-2012	Option d
Q.No: 3 TIFR-2013	Option c
Q.No: 4 TIFR-2014	Option c
Q.No: 5 TIFR-2014	Option c
Q.No: 6 TIFR-2014	Option d
Q.No: 7 TIFR-2014	Option d
Q.No: 8 TIFR-2015	Option c
Q.No: 9 TIFR-2015	Option b
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Q.No: 14 TIFR-2018	Option b
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Q.No: 16 TIFR-2020	Option b
Q.No: 17 TIFR-2020	Option c
Q.No: 18 TIFR-2012	Option e
Q.No: 19 TIFR-2021	Option a
Q.No: 20 TIFR-2021	Option a
Q.No: 21 TIFR-2021	Option a
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Q.No: 24 TIFR-2023	Option b
Q.No: 25 TIFR-2024	Option 1
Q.No: 26 TIFR-2024	Option a
Q.No: 27 TIFR-2024	Option c
Q.No: 28 TIFR-2025	-
Q.No: 29 TIFR-2025	Option A
Q.No: 30 TIFR-2025	Option A
Q.No: 31 TIFR-2026	Option A
Q.No: 32 TIFR-2026	Option A
Q.No: 33 TIFR-2026	Option A
Q.No: 34 TIFR-2026	Option A

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