Basics, TISE, 1D Box and LHO-GATE

https://snboseplc.com/basics-tise-1d-box-and-lho/

Q.No:1 GATE-2012

A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is CORRECT?

$$V(x) = \begin{cases} \infty & x < 0 \\ \frac{1}{2}kx^2 & x \geq 0 \end{cases}$$

1. (A) The parity of the first excited state is even
2. (B) The parity of the ground state is even
3. (C) The ground state energy is $\frac{1}{2}\hbar\omega$
4. (D) The first excited state energy is $\frac{7}{2}\hbar\omega$

Q.No:2 GATE-2014

The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength $1500nm$, and it is p_B when it emits a photon of visible wavelength $500nm$. The ratio $\frac{p_A}{p_B}$ is

1. (A) $1:1$
2. (B) $1:\sqrt{3}$
3. (C) $1:3$
4. (D) $3:2$

Q.No:3 GATE-2014

A one dimensional harmonic oscillator is in the superposition of number states, $|n\rangle$, given by

$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle.$$

The average energy of the oscillator in the given state is ..... $\hbar\omega$.

Q.No:4 GATE-2015

Suppose a linear harmonic oscillator of frequency ω and mass m is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{i\frac{\pi}{2}}|\psi_1\rangle\right]$ at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\psi|x|\psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is .....

Q.No:5 GATE-2016

The total power emitted by a spherical black body of radius R at a temperature T is P_1. Let P_2 be the total power emitted by another spherical black body of radius $R/2$ kept at temperature $2T$. The ratio, P_1/P_2 is .....
(Give your answer upto two decimal places)

Q.No:6 GATE-2016

The state of a system is given by

$$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$$

where $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is .....
(Give your answer upto two decimal places)

Q.No:7 GATE-2017

The Compton wavelength of a proton is ..... fm. (up to two decimal places). $m_p = 1.67 \times 10^{-27}$ kg, $h = 6.626 \times 10^{-34}$ Js, $e = 1.602 \times 10^{-19}$ C, $c = 3 \times 10^8 ms^{-1}$

Q.No:8 GATE-2017

Consider a one-dimensional potential well of width $3nm$. Using the uncertainty principle ($\Delta x.\Delta p \geq \hbar/2$), an estimate of the minimum depth of the well such that it has at least one bound state for an electron is ($m_e = 9.31\times10^{-31}kg, h = 6.626\times10^{-34}Js, e = 1.602\times10^{-19}C$):

1. (A) $1\mu eV$
2. (B) $1meV$
3. (C) $1eV$
4. (D) $1MeV$

Q.No:9 GATE-2017

The integral $\int_0^\infty x^2 e^{-x^2} dx$ is equal to ..... (up to two decimal places).

Q.No:10 GATE-2018

Given the following table,

Group I	Group II
P: Stern-Gerlach experiment	**1**: Wave nature of particles
Q: Zeeman effect	**2**: Quantization of energy of electrons in the atoms
R: Frank-Hertz experiment	**3**: Existence of electron spin
S: Davisson-Germer experiment	**4**: Space quantization of angular momentum

Figure 1: Table for Q.No:10

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which one of the following correctly matches the experiments from Group I to their inferences in Group II?

1. (A) P-2, Q-3, R-4, S-1
2. (B) P-3, Q-4, R-2, S-1
3. (C) P-1, Q-3, R-2, S-4
4. (D) P-2, Q-1, R-4, S-3

Q.No:11 GATE-2018

A two-state quantum system has energy eigenvalues $\pm\epsilon$ corresponding to the normalized states $|\psi_\pm\rangle$. At time $t = 0$, the system is in quantum state $\frac{1}{\sqrt{2}}[|\psi_+\rangle + |\psi_-\rangle]$. The probability that the system will be in the same state at $t = h/(6\epsilon)$ is ..... (up to two decimal places).

Q.No:12 GATE-2018

Two solid spheres A and B have same emissivity. The radius of A is four times the radius of B, and temperature of A is twice the temperature of B. The ratio of the rate of heat radiated from A to that from B is .....

Q.No:13 GATE-2019

The wave function $\psi(x)$ of a particle is as shown below

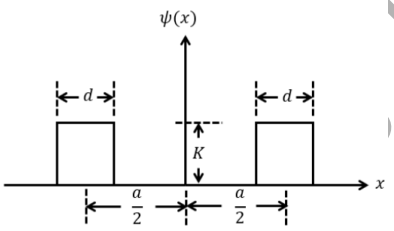


Figure 2: Figure for Q.No:13

Here K is a constant, and $a > d$. The position uncertainty $\langle\Delta x\rangle$ of the particle is

1. (A) $\sqrt{\frac{a^2+3d^2}{12}}$
2. (B) $\sqrt{\frac{3a^2+d^2}{12}}$
3. (C) $\sqrt{\frac{d^2}{6}}$
4. (D) $\sqrt{\frac{d^2}{24}}$

Q.No:14 GATE-2019

Consider the motion of a particle along the x-axis in a potential $V(x) = F|x|$. Its ground state energy E_0 is estimated using the uncertainty principle. Then E_0 is proportional to

1. (A) $F^{\frac{1}{3}}$
2. (B) $F^{\frac{1}{2}}$
3. (C) $F^{\frac{2}{5}}$
4. (D) $F^{\frac{2}{3}}$

Q.No:15 GATE-2020

A quantum particle is subjected to the potential

$$V(x) = \begin{cases} \infty, & x \leq -\frac{a}{2} \\ 0, & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & x \geq \frac{a}{2}. \end{cases}$$

The ground state wave function of the particle is proportional to

1. (A) $\sin\left(\frac{\pi x}{2a}\right)$
2. (B) $\sin\left(\frac{\pi x}{a}\right)$
3. (C) $\cos\left(\frac{\pi x}{2a}\right)$
4. (D) $\cos\left(\frac{\pi x}{a}\right)$

Q.No:16 GATE-2020

Let $\hat{a}$ and $\hat{a}^\dagger$, respectively denote the lowering and raising operators of a one-dimensional simple harmonic oscillator. Let $|n\rangle$ be the energy eigenstate of the simple harmonic oscillator. Given that $|n\rangle$ is also an eigenstate of $\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}$, the corresponding eigenvalue is

1. (A) $n(n-1)$
2. (B) $n(n+1)$
3. (C) $(n+1)^2$
4. (D) n^2

Q.No:17 GATE-2021

A light source having its intensity peak at the wavelength $289.8nm$ is calibrated as 10000 K which is the temperature of an equivalent black body radiation. Considering the same calibration, the temperature of light source (in K) having its intensity peak at the wavelength $579.6nm$ (rounded off to the nearest integer) is .....

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Q.No:18 GATE-2021

Consider a state described by $\psi(x,t) = \psi_2(x,t) + \psi_4(x,t)$, where $\psi_2(x,t)$ and $\psi_4(x,t)$ are respectively the second and fourth normalized harmonic oscillator wave functions and ω is the angular frequency of the harmonic oscillator. The wave function $\psi(x, t = 0)$ will be orthogonal to $\psi(x,t)$ at time t equal to

1. (A) $\frac{\pi}{2\omega}$
2. (B) $\frac{\pi}{\omega}$
3. (C) $\frac{\pi}{4\omega}$
4. (D) $\frac{\pi}{6\omega}$

Q.No:19 GATE-2022

The wavefunction of a particle in a one-dimensional infinite well of size $2a$ at a certain time is

$$\psi(x) = \frac{1}{\sqrt{6a}}\left[\sqrt{2}\sin\left(\frac{\pi x}{a}\right)\right] + \frac{1}{\sqrt{6a}}\left[\sqrt{3}\cos\left(\frac{\pi x}{2a}\right) + \cos\left(\frac{3\pi x}{2a}\right)\right]$$

Probability of finding the particle in $n = 2$ state at that time is ————————- % (Round off to the nearest integer)

Q.No:20 GATE-2023

Which one of the following options is the most appropriate match between the items given in Column 1 and Column 2 ?

Column 1	Column 2
(i) Visible light	P. Transition between core energy levels of atoms
(ii) X-rays	Q. Transition between nuclear energy levels
(iii) Gamma rays	R. Pair production
(iv) Thermal neutrons	S. Crystal structure determination
	T. Photoelectric effect

Figure 3: Table for Q.No:20

1. (A) (i) - T; (ii) - P,S,T; (iii) - Q,R; (iv) - S
2. (B) (i) - P,T; (ii) - S; (iii) - R,S; (iv) - S,T
3. (C) (i) - T; (ii) - R,S; (iii) - Q,R; (iv) - S
4. (D) (i) - S,T; (ii) - P,S; (iii) - R,T; (iv) - S

Q.No:22 GATE-2024

The wavefunction of a particle in an infinite one-dimensional potential well at time t is

$$\psi(x,t) = \sqrt{\frac{2}{3}}e^{-iE_1 t/\hbar}\psi_1(x) + \frac{1}{\sqrt{6}}e^{i\pi/6}e^{-iE_2 t/\hbar}\psi_2(x) + \frac{1}{\sqrt{6}} \times e^{i\pi/4}e^{-iE_3 t/\hbar}\psi_3(x)$$

where ψ_1, ψ_2 and ψ_3 are the normalized ground state, the normalized first excited state and the normalized second excited state, respectively. E_1, E_2 and E_3 are the eigen-energies corresponding to ψ_1, ψ_2 and ψ_3, respectively. The expectation value of energy of the particle in state $\psi(x,t)$ is

1. 1) $\frac{17}{6}E_1$
2. 2) $\frac{2}{3}E_1$
3. 3) $\frac{3}{2}E_1$
4. 4) $14E_1$

Q.No:23 GATE-2024

A particle of mass m is moving in the potential

$$V(x) = \begin{cases} V_0 + \frac{1}{2}m\omega_0^2 x^2, & x > 0, \\ \infty, & x \le 0, \end{cases}$$

Figures P, Q, R and S show different combinations of the values of ω_0 and V_0.

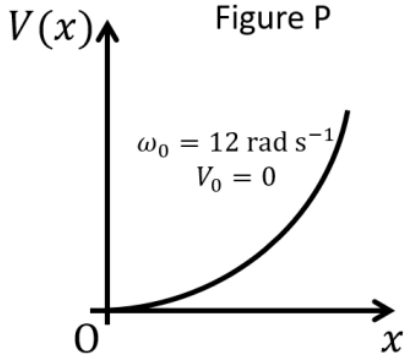


Figure 4: Figure for Q.No:23 (P)

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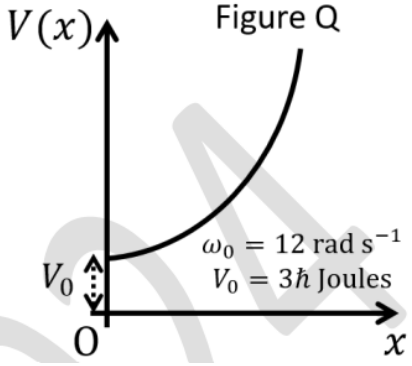


Figure 5: Figure for Q.No:23 (Q)

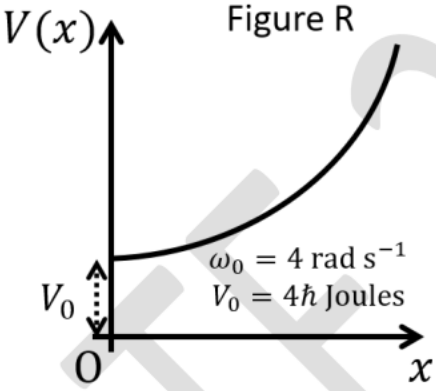


Figure 6: Figure for Q.No:23 (R)

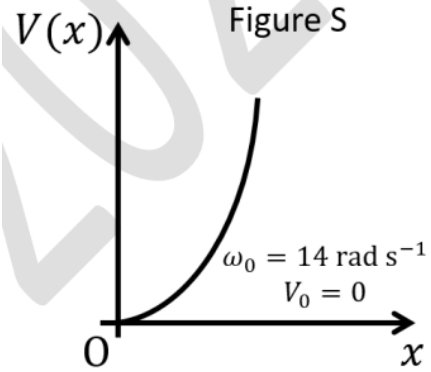


Figure 7: Figure for Q.No:23 (S)

$E_j^{(P)}$, $E_j^{(Q)}$, $E_j^{(R)}$ and $E_j^{(S)}$ with $j = 0, 1, 2, ...$ are the eigen-energies of the j-th level for the potentials shown in figures P, Q, R and S, respectively. Which of the statement is/are true?

1. 1) $E_0^{(P)} = E_0^{(Q)}$
2. 2) $E_0^{(Q)} = E_0^{(S)}$
3. 3) $E_0^{(P)} = E_1^{(R)}$
4. 4) $E_0^{(R)} \neq E_0^{(Q)}$

Q.No:24 GATE-2024

The wavefunction for a particle is given by the form $e^{-(i\alpha x+\beta)}$, where α and β are real constants. In which one of the following potentials $V(x)$, the particle is moving?

1. 1) $V(x) \propto \alpha^2 x^2$
2. 2) $V(x) \propto e^{-\alpha x}$
3. 3) $V(x) = 0$
4. 4) $V(x) \propto \sin(\alpha x)$

Q.No:25 GATE-2025

A particle of mass m is in a potential $V(x) = \frac{1}{2}m\omega^2 x^2$ for $x > 0$ and $V(x) = \infty$ for $x \leq 0$, where ω is the angular frequency. The ratio of the energies corresponding to the lowest energy level to the next higher level is

1. 1) $\frac{3}{7}$
2. 2) $\frac{1}{2}$
3. 3) $\frac{1}{3}$
4. 4) $\frac{3}{5}$

Q.No:26 GATE-2025

Which of the following option(s) is/are correct for photons?

1. 1) Its rest mass is zero, but its energy is non-zero
2. 2) It carries non-zero linear momentum
3. 3) It carries zero spin angular momentum
4. 4) It has two linearly independent states of polarization

Q.No:27 GATE-2025

Let $|m\rangle$ and $|n\rangle$ denote the energy eigenstates of a one-dimensional simple harmonic oscillator. The position and momentum operators are $\hat{X}$ and $\hat{P}$, respectively. The matrix element $\langle m|\, \hat{P}\hat{X}\, |n\rangle$ is non-zero when

1. 1) $m = n \pm 2$ only
2. 2) $m = n$ or $m = n \pm 2$
3. 3) $m = n \pm 3$ only
4. 4) $m = n \pm 1$ only

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Question No.	Entrance with year	Answer
Q.No:1	GATE-2012	Option D
Q.No:2	GATE-2014	Option C
Q.No:3	GATE-2014	3.2 - 3.3 (Numerical)
Q.No:4	GATE-2015	0 (Numerical)
Q.No:5	GATE-2016	0.25 (Numerical)
Q.No:6	GATE-2016	0.27 - 0.29 (Numerical)
Q.No:7	GATE-2017	1.30 - 1.34 fm (Numerical)
Q.No:8	GATE-2017	Option B
Q.No:9	GATE-2017	0.43 - 0.45 (Numerical)
Q.No:10	GATE-2018	Option B
Q.No:11	GATE-2018	0.25 (Numerical)
Q.No:12	GATE-2018	256 (Numerical)
Q.No:13	GATE-2019	Option B
Q.No:14	GATE-2019	Option D
Q.No:15	GATE-2020	Option D
Q.No:16	GATE-2020	Option A
Q.No:17	GATE-2021	5000 K (Numerical)
Q.No:18	GATE-2021	Option A
Q.No:19	GATE-2022	33 - 34% (Numerical)
Q.No:20	GATE-2023	Option A
Q.No:22	GATE-2024	Option 1
Q.No:23	GATE-2024	Options 2, 3, 4
Q.No:24	GATE-2024	Option 3
Q.No:25	GATE-2025	Option 1
Q.No:26	GATE-2025	Options 1, 2, 4
Q.No:27	GATE-2025	Option 2

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