

Dirac Well Finite Well and Scattering State Problems GATE CSIR

<https://snboseplc.com/dirac-well-finite-well-and-scattering-state-problems-gate-csir/>

GATE Questions

Q.No:1 GATE-2012

The wavefunction of a particle moving in free space is given by,

$$\psi = e^{ikx} + 2e^{-ikx}$$

The energy of the particle is

1. $\frac{5\hbar^2 k^2}{2m}$
2. $\frac{3\hbar^2 k^2}{4m}$
3. $\frac{\hbar^2 k^2}{2m}$
4. $\frac{\hbar^2 k^2}{m}$

Q.No:2 GATE-2012

The wavefunction of a particle moving in free space is given by,

$$\psi = e^{ikx} + 2e^{-ikx}$$

The probability current density for the real part of the wavefunction is

1. 1
2. $\frac{\hbar k}{m}$
3. $\frac{\hbar k}{2m}$
4. 0

Q.No:3 GATE-2015

The dispersion relation for phonons in a one dimensional monatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by,

$$\omega(k) = \sqrt{\frac{2C}{M}} [1 - \cos(ka)]$$

where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes ($\lambda \gg a$), the ratio of the phase velocity to the group velocity is _____

Q.No:4 GATE-2016

A particle of mass m and energy E , moving in the positive x direction, is incident on a step potential at $x = 0$, as indicated in the figure. The height of the potential is V_0 , where $V_0 > E$. At $x = x_0$, where $x_0 > 0$, the probability of finding the electron is $1/e$ times the probability of finding it at $x = 0$. If $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$, the value of x_0 is

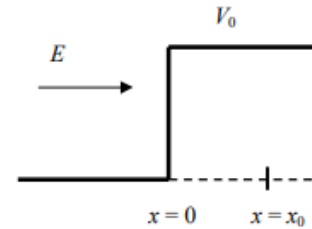


Figure 1: Potential step

1. $\frac{2}{\alpha}$
2. $\frac{1}{\alpha}$
3. $\frac{1}{2\alpha}$
4. $\frac{1}{4\alpha}$

Q.No:5 GATE-2017

A free electron of energy 1 eV is incident upon a one-dimensional finite potential step of height 0.75 eV. The probability of its reflection from the barrier is _____ (up to two decimal places).

Q.No:6 GATE-2019

Consider a potential barrier $V(x)$ of the form:

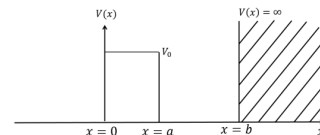


Figure 2: Potential barrier

where V_0 is a constant. For particles of energy $E < V_0$ incident on this barrier from the left, which of the following schematic diagrams best represents the probability density $|\psi(x)|^2$ as a function of x .

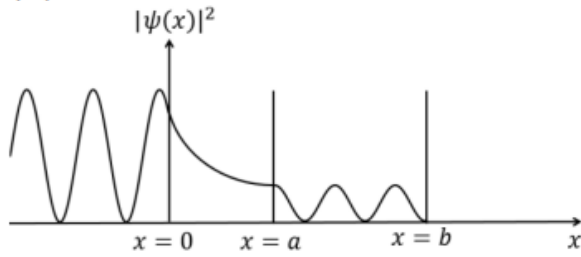


Figure 3: (A)

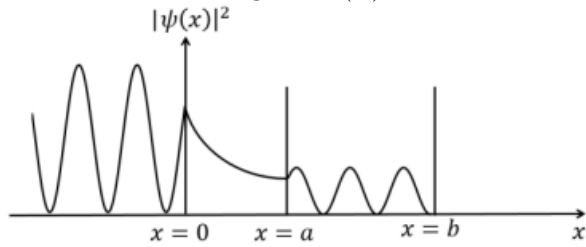


Figure 4: (B)

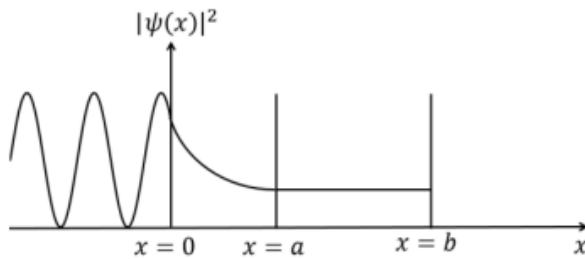


Figure 5: (C)

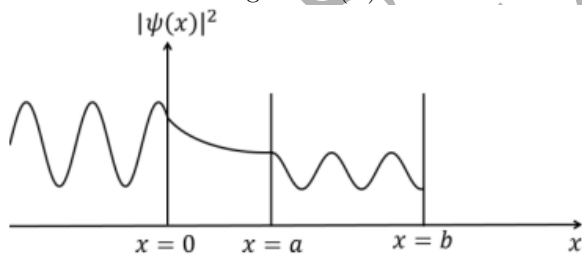


Figure 6: (D)

Q.No: 7 GATE-2024

A particle is subjected to a potential

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ V_0, & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

Here, $a > 0$ and $b > a$. If the energy of the particle $E < V_0$, which one of the following schematics is a valid quantum mechanical wavefunction (Ψ) for the system?

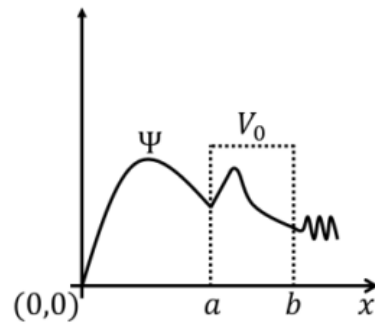


Figure 7: Option 1

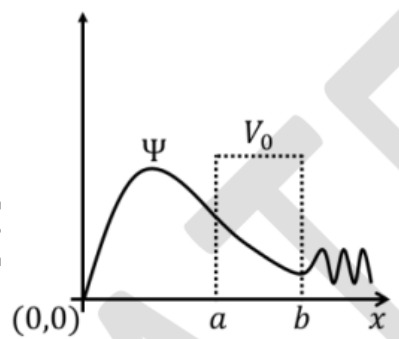


Figure 8: Option 2

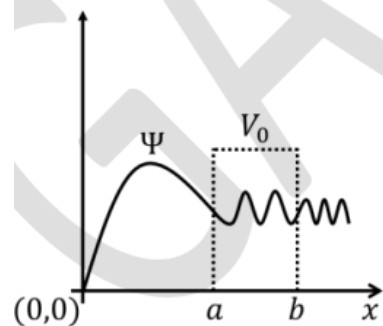


Figure 9: Option 3

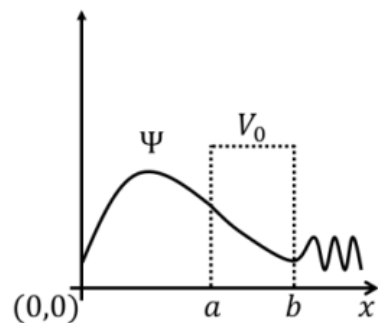


Figure 10: Option 4

Q.No: 8 GATE-2025

Which one of the following is correct for the phase velocity v_p and group velocity v_g ? (c is the speed of light in vacuum)

1. For matter waves in the relativistic case, $v_p v_g > c^2$
2. For electromagnetic waves in a medium, v_p represents the speed with which energy propagates
3. For electromagnetic waves in a medium, both v_p and v_g can be more than c
4. For matter waves in free space, $v_p \neq v_g$

CSIR Questions**Q.No:1 CSIR-June-2016**

A particle of mass m moves in one dimension under the influence of the potential $V(x) = -\alpha\delta(x)$, where α is a positive constant. The uncertainty in the product $(\Delta x)(\Delta p)$ in its ground state is

1. $2\hbar$
2. $\hbar/2$
3. $\hbar/\sqrt{2}$
4. $\sqrt{2}\hbar$

Q.No:2 CSIR-Dec-2016

A particle in one dimension is in a potential $V(x) = A\delta(x - a)$. Its wavefunction $\psi(x)$ is continuous everywhere. The discontinuity in $\frac{d\psi}{dx}$ at $x = a$ is

1. $\frac{2m}{\hbar^2} A\psi(a)$
2. $A(\psi(a) - \psi(-a))$
3. $\frac{\hbar^2}{2m} A$
4. 0

Q.No:3 CSIR-June-2017

Consider a potential barrier A of height V_0 and width b , and another potential barrier B of height $2V_0$ and the same width b . The ratio T_A/T_B of tunnelling probabilities T_A and T_B , through barriers A and B respectively, for a particle of energy $V_0/100$, is best approximated by

1. $\exp \left[(\sqrt{1.99} - \sqrt{0.99}) \sqrt{8mV_0 b^2 / \hbar^2} \right]$
2. $\exp \left[(\sqrt{1.98} - \sqrt{0.98}) \sqrt{8mV_0 b^2 / \hbar^2} \right]$
3. $\exp \left[(\sqrt{2.99} - \sqrt{0.99}) \sqrt{8mV_0 b^2 / \hbar^2} \right]$
4. $\exp \left[(\sqrt{2.98} - \sqrt{0.98}) \sqrt{8mV_0 b^2 / \hbar^2} \right]$

Q.No:4 CSIR-Dec-2017

The normalized wavefunction in the momentum space of a particle in one dimension is $\phi(p) = \frac{\alpha}{p^2 + \beta^2}$, where α and β are real constants. The uncertainty Δx in measuring its position is

1. $\sqrt{\pi} \frac{\hbar\alpha}{\beta^2}$

2. $\sqrt{\pi} \frac{\hbar \alpha}{\beta^3}$

3. $\frac{\hbar}{\sqrt{2}\beta}$

4. $\sqrt{\frac{\pi}{\beta}} \frac{\hbar \alpha}{\beta}$

Q.No:5 CSIR-June-2019

A particle of mass m and energy $E > 0$, in one dimension is scattered by the potential shown below.

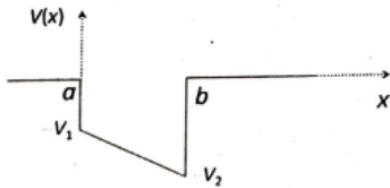


Figure 11: Scattering potential

Which one of the following figures best represents the reflected wavefunction?

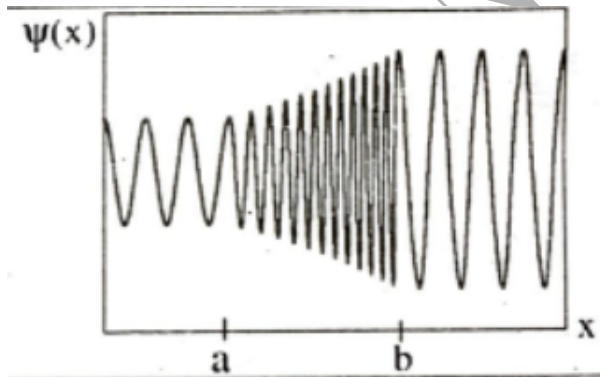


Figure 12: (1)

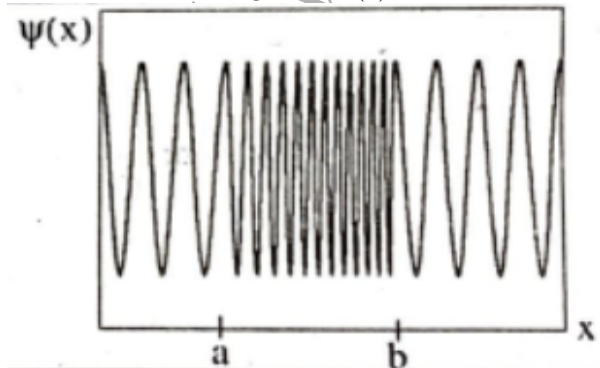


Figure 13: (2)

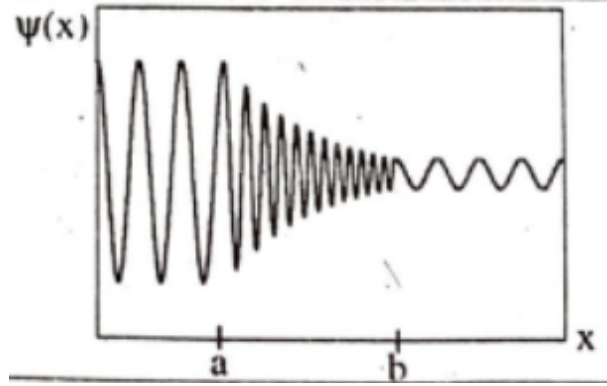


Figure 14: (3)

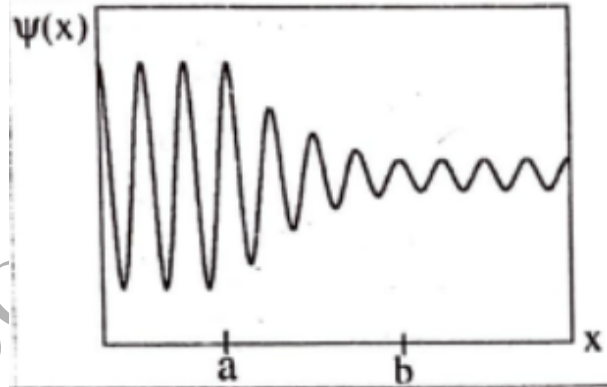


Figure 15: (4)

Q.No:6 CSIR-June-2019

The wave number k and the angular frequency ω of a wave are related by the dispersion relation $\omega^2 = \alpha k + \beta k^3$, where α and β are positive constants. The wave number for which the phase velocity equals the group velocity, is

1. $3\sqrt{\frac{\alpha}{\beta}}$

2. $\sqrt{\frac{\alpha}{\beta}}$

3. $\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}$

4. $\frac{1}{3}\sqrt{\frac{\alpha}{\beta}}$

Q.No:7 CSIR-June-2019

The operator $x \frac{d}{dx} \delta(x)$, where $\delta(x)$ is the Dirac delta function, acts on the space of real-valued square-integrable functions on the real line. This operator is equivalent to

1. $-\delta(x)$

2. $\delta(x)$

3. x

4. 0

Q.No:8 Assam CSIR-Dec-2019

The wavefunction of a particle of mass m , in a potential $V(x)$ in one dimension is $\psi(x) = Ae^{-\beta|x|}$, where $\beta > 0$ and A are constants. If this wavefunction is an energy eigenfunction, then a possible form of the potential $V(x)$ is

1. $\frac{\hbar^2 \beta}{m |x|}$
2. $-\frac{\hbar^2 \beta}{m |x|}$
3. $\frac{\hbar^2 \beta}{m} \delta(x)$
4. $-\frac{\hbar^2 \beta}{m} \delta(x)$

Q.No:9 CSIR-June-2020

For the one dimensional potential wells A, B and C, as shown in the figure, let E_A, E_B and E_C denote the ground state energies of a particle, respectively.

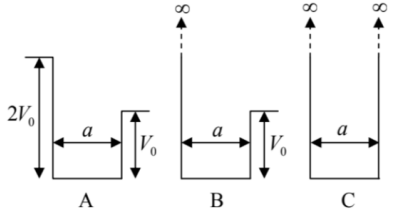


Figure 16: Potential wells A, B, and C

The correct ordering of the energies is

1. $E_C > E_B > E_A$
2. $E_A > E_B > E_C$
3. $E_B > E_C > E_A$
4. $E_B > E_A > E_C$

Q.No:10 CSIR-June-2020

A quantum particle in a one-dimensional infinite potential well, with boundaries at 0 and a , is perturbed by adding $H' = \epsilon \delta(x - \frac{a}{2})$ to the initial Hamiltonian. The correction to the energies of the ground and the first excited states (to first order in ϵ) are respectively

1. 0 and 0
2. $2\epsilon/a$ and 0
3. 0 and $2\epsilon/a$
4. $2\epsilon/a$ and $2\epsilon/a$

Q.No:11 CSIR Sep-2022

If the expectation value of the momentum of a particle in one dimension is zero, then its (box-normalizable) wavefunction may be of the form

1. $\sin(kx)$
2. $e^{ikx} \sin(kx)$
3. $e^{ikx} \cos(kx)$
4. $\sin(kx) + e^{ikx} \cos(kx)$

Q.No:12 CSIR Sep-2022

The energy/energies E of the bound state(s) of a particle of mass m in one dimension in the potential

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ -V_0, & 0 \leq x \leq a \\ 0, & x \geq a \end{cases}$$

is given by the equation:

1. $\cot^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = \frac{E-V_0}{E}$
2. $\tan^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = -\frac{E}{E+V_0}$
3. $\cot^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = -\frac{E}{E+V_0}$
4. $\tan^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = \frac{E-V_0}{E}$

Q.No:13 CSIR Dec-2023

A quantum particle of mass m is moving in a one dimensional potential

$$V(x) = V_0 \theta(x) - \lambda \delta(x),$$

where V_0 and λ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E \gg V_0 > \lambda$ and $\sqrt{2mE} \gg \frac{V_0 \hbar}{\lambda}$ is

1. $\frac{V_0^2}{4E^2}$
2. $\frac{V_0^2}{8E^2}$
3. $\frac{m\lambda^2}{2E\hbar^2}$
4. $\frac{m\lambda^2}{4E\hbar^2}$

Q.No:14 CSIR June-2024

A particle of energy E is scattered off a one-dimensional potential $\lambda\delta(x)$, where λ is a real positive constant, with a transmission amplitude t_+ . In a different experiment, the same particle is scattered off another one-dimensional potential $-\lambda\delta(x)$, with a transmission amplitude t_- . In the limit $E \rightarrow 0$, the phase difference between t_+ and t_- is

1. $\pi/2$
2. π
3. 0
4. $3\pi/2$

Q.No:15 CSIR June-2024

Using a normalized trial wavefunction $\psi(x) = \sqrt{\alpha}e^{-\alpha|x|}$ (α is a positive real constant) for a particle of mass m in the potential $V(x) = -\lambda\delta(x)$, ($\lambda > 0$), the estimated ground state energy is

1. $-\frac{m\lambda^2}{\hbar^2}$
2. $\frac{m\lambda^2}{\hbar^2}$
3. $\frac{m\lambda^2}{2\hbar^2}$
4. $-\frac{m\lambda^2}{2\hbar^2}$

Q.No:16 CSIR Dec-2024

A particle of mass m is bound in one dimension by the potential $V(x) = V_0\delta(x)$ with $V_0 < 0$. If the probability of finding it in the region $|x| < a$ is 0.25, then a is

1. $\frac{\hbar^2}{4mV_0} \ln \frac{3}{4}$
2. $\frac{\hbar^2}{2mV_0} \ln \frac{3}{4}$
3. $\frac{\hbar^2}{4mV_0} \ln \frac{1}{4}$
4. $\frac{\hbar^2}{2mV_0} \ln \frac{1}{4}$

Q.No:17 CSIR Dec-2024

A particle of mass m , moving in one dimension is subjected to the potential

$$V(x) = \begin{cases} V_0\delta(x-a) & 0 < x < 2a \\ \infty & \text{otherwise} \end{cases}$$

The energy eigenvalues E satisfy

1. $\tan\left(\frac{a\sqrt{2mE}}{\hbar}\right) = \frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$
2. $\tanh\left(\frac{a\sqrt{2mE}}{\hbar}\right) = \frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$
3. $\tan\left(\frac{a\sqrt{2mE}}{\hbar}\right) = -\frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$
4. $\tanh\left(\frac{a\sqrt{2mE}}{\hbar}\right) = -\frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$

Q.No:18 CSIR June-2025

The probability density of a free particle of mass m at time $t = 0$ is given by

$$A \exp\left(-\frac{x^2}{2\sigma^2(0)}\right)$$

At $t > 0$, its probability density is proportional to

$$\exp\left(-\frac{x^2}{2\sigma^2(t)}\right)$$

where $\sigma^2(t)$ is

1. $\sigma^2(0) + \frac{\hbar^2 t^2}{\sigma^2(0)m^2}$
2. $\sigma^2(0) + \frac{\hbar^2 t^2}{4\sigma^2(0)m^2}$
3. $\sigma^2(0) + \frac{4\hbar^2 t^2}{\sigma^2(0)m^2}$
4. $\sigma^2(0) + \frac{2\hbar^2 t^2}{\sigma^2(0)m^2}$

S. N. Bose Physics Learning Centre

Website: <https://snboseplc.com/>

Email: subrahmanya@snboseplc.com

Question No.	Entrance with year	Answer
Q.No:1	GATE-2012	Option C
Q.No:2	GATE-2012	Option D
Q.No:3	GATE-2015	1
Q.No:4	GATE-2016	Option C
Q.No:5	GATE-2017	0.10–0.12
Q.No:6	GATE-2019	Option A
Q.No:7	GATE-2024	Option 2
Q.No:8	GATE-2025	Option 4
Q.No:1	CSIR-June-2016	Option 3
Q.No:2	CSIR-Dec-2016	Option 1
Q.No:3	CSIR-June-2017	Option 1
Q.No:4	CSIR-Dec-2017	Option 3
Q.No:5	CSIR-June-2019	Option 3
Q.No:6	CSIR-June-2019	Option 2
Q.No:7	CSIR-June-2019	Option 1
Q.No:8	CSIR-Dec-2019 (Assam)	Option 4
Q.No:9	CSIR-June-2020	Option a
Q.No:10	CSIR-June-2020	Option b
Q.No:11	CSIR-Sep-2022	Option 1
Q.No:12	CSIR-Sep-2022	Option 3
Q.No:13	CSIR-Dec-2023	Option 3
Q.No:14	CSIR-June-2024	Option 2
Q.No:15	CSIR-June-2024	Option 4
Q.No:16	CSIR-Dec-2024	Option 2
Q.No:17	CSIR-Dec-2024	Option 3
Q.No:18	CSIR-June-2025	Option 2