Dirac Well Finite Well and Scattering State Problems JEST TIFR

https://snboseplc.com/dirac-well-finite-well-and-scattering-state-problems-jest-tifr/

JEST Questions

Q.No:1 JEST-2012

The wave function of a free particle in one dimension is given by

$$\psi(x) = A\sin x + B\sin 3x.$$

Then $\psi(x)$ is an eigenstate of

1. the position operator
2. the Hamiltonian
3. the momentum operator
4. the parity operator

Q.No:2 JEST-2014

Consider a square well of depth $-V_0$ and width a with $V_0 a$ fixed. Let $V_0 \to \infty$ and $a \to 0$. This potential well has

1. No bound states
2. 1 bound state
3. 2 bound states
4. Infinitely many bound states

Q.No:3 JEST-2020

A free particle of energy E, characterized by a plane wave of wavelength λ enters a region of constant potential $-V$ (where $E > V > 0$). Within the region of the potential, the wavelength of the particle is $\lambda/2$. The ratio V/E is:

1. $-1/3$
2. -3
3. 3
4. $1/3$

Q.No:4 JEST-2021

A quantum particle is moving in one dimension between rigid walls at $x = -L$ and $x = L$, under the influence of a potential (see figure). The potential has the uniform value V_0 between $-a < x < a$, and is 0 otherwise. Which one of the following graphs qualitatively represent the ground state wavefunction of this system? (You can assume that $a \ll L$ and $V_0 \gg \pi^2/8mL^2$).

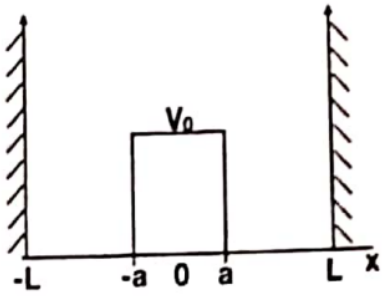


Figure 1: Potential well

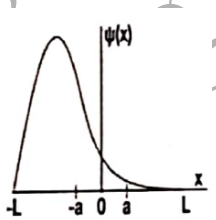


Figure 2: (a)

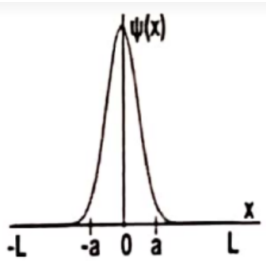


Figure 3: (b)

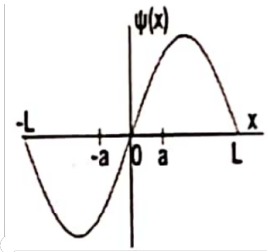


Figure 4: (c)

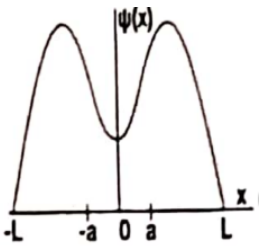


Figure 5: (d)

Q.No:5 JEST-2022

A particle is confined in an infinite potential well of the form given below.

$$V(x) = \begin{cases} 4V_0 x(1-x), & \forall 0 \le x \le 1 \\ \infty, & \text{otherwise} \end{cases}$$

If the particle has energy $E \ge V_0$, which of the following could be the form of its wavefunction?

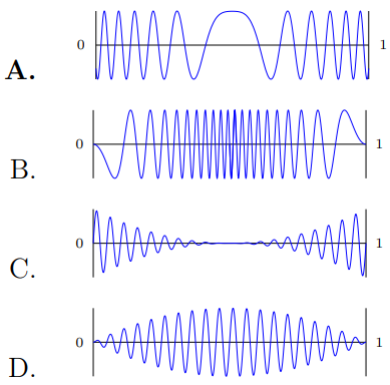


Figure 6: Wavefunction options

Q.No: 7 JEST-2023

Consider a quantum particle incident from the left on a step potential given by $V_0\theta(x)$, with energy $E(> V_0)$; here $\theta(x)$ is the unit step function. The scattering state of the particle is given by

$$\psi(x) = \begin{cases} \exp\frac{ipx}{\hbar} + r\exp\frac{-ipx}{\hbar}, & x < 0 \\ t\exp\frac{ip'x}{\hbar} & x > 0 \end{cases}$$

Figure 7: Scattering state

where p and p' are the momenta of the particle corresponding to the energy E. Which of the following is true?

1. $|r|^2+\frac{p'}{p}|t|^2= 1$
2. $|r|^2+|t|^2= 1$
3. $|r|^2+\frac{p}{p'}|t|^2= 1$
4. $r + t = 1$

Q.No: 8 JEST-2023

Consider a free particle in one dimension described by the wavefunction:

$$\psi(x, t = 0) = A\exp\left(\frac{ipx}{\hbar}\right) + B\exp\left(\frac{-ipx}{\hbar}\right)$$

The probability current density corresponding to $\psi(x, t)$ at a later time t is:

1. $\frac{p(|A|^2-|B|^2)}{m}\cos\left(\frac{p^2}{2m\hbar}t\right)$
2. $\frac{p(|A|^2-|B|^2)}{m}$
3. $\frac{p(|A|^2+|B|^2)}{m}$
4. $\frac{p(|A|^2+|B|^2)}{m}\cos\left(\frac{p^2}{2m\hbar}t\right)$

Q.No : 9 JEST-2024

A particle with energy $E > 0$ is incident from the right ($x > 0$) on a one-dimensional potential composed of a delta-function barrier at $x = 0$ and a hard wall at $x = -a$:

$$V(x) = \begin{cases} \alpha\delta(x), & x > -a \\ \infty, & x \leq -a \end{cases}$$

where $\alpha > 0$ and $a > 0$. Let us define $k^2 = \frac{2mE}{\hbar^2}$ and the dimensionless quantities: $\xi = \kappa a$ and $\beta = \frac{\hbar^2}{2m\alpha a}$. For some energy E the particle reflects from the barrier without any phase shift. Which of the following transcendental equations determines this energy? Note that in the presence of the delta function barrier, the derivative of the wave function has a discontinuity at $x = 0$: $\psi'(0^+) - \psi'(0^-) = \frac{\psi(0)}{\beta a}$.

1. $\tan\xi = -\beta\xi$
2. $\tan\xi = \beta\xi$
3. $\tanh\xi = \beta\xi$
4. $\tanh\xi = -\beta\xi$

Q.No : 10 JEST-2025

Suppose the wave function of a free particle in one dimension obeys

$$\frac{d^2\psi}{dx^2} = -4\psi$$

in units where $\hbar = 1$. What is the magnitude of the momentum of the particle?

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TIFR Questions

Q.No: 1 TIFR-2012

A particle in a one-dimensional potential has the wavefunction

$$\psi(x) = \frac{1}{\sqrt{a}} \exp\left(\frac{-|x|}{a}\right)$$

where a is a constant. It follows that for a positive constant V_0, the potential $V(x) =$

1. $V_0 x^2$
2. $V_0|x|$
3. $-V_0\delta(x)$
4. $-V_0/|x|$

Q.No: 2 TIFR-2014

A particle is confined to a one-dimensional box of length L. If a vanishingly thin but strongly repulsive partition is introduced in the exact centre of the box, and the particle is allowed to come to its ground state, then the probability density for finding the particle will appear as

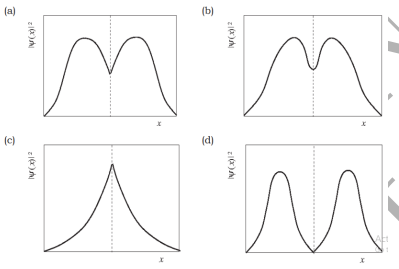


Figure 8: Probability density options

Q.No: 3 TIFR-2015

It is required to construct the quantum theory of a particle of mass m moving in one dimension x under the influence of a constant force F. The characteristic length-scale in this problem is

1. $\dfrac{\hbar}{mF}$
2. $\left(\dfrac{\hbar^2}{mF}\right)$
3. $\left(\dfrac{\hbar}{m^2F}\right)$
4. $\dfrac{mF}{\hbar^2}$

Q.No:4 TIFR-2016

A particle is confined inside a one-dimensional potential well $V(x)$, as shown on the right.

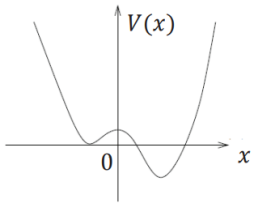


Figure 9: Potential well

One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is

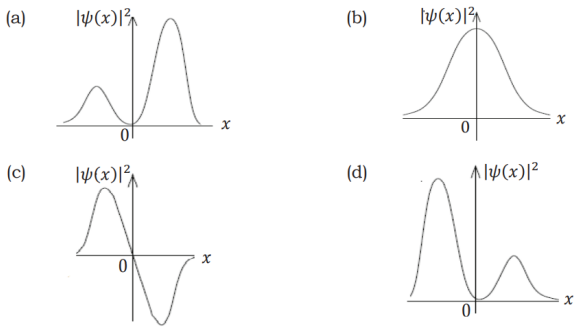


Figure 10: Probability distribution options

Q.No: 5 TIFR-2017

Consider the 1-D asymmetric double-well potential $V(x)$ as sketched below.

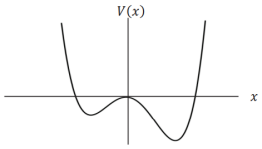


Figure 11: Asymmetric double-well potential

The probability distribution $p(x)$ of a particle in the ground state of this potential is best represented by

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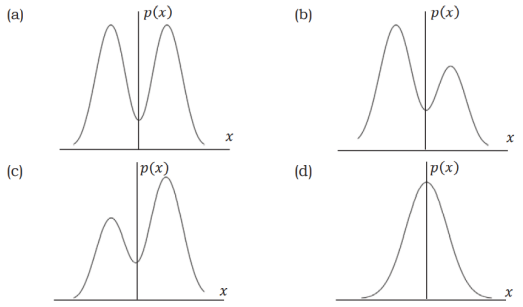


Figure 12: Probability distribution options

Q.No :6 TIFR-2019

A particle of mass m, moving in one dimension, satisfies the modified Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + i\hbar u\frac{d\psi}{dx} = i\hbar\frac{d\psi}{dt}$$

where u is the velocity of the substrate. If, now, this particle is treated as a Gaussian wave packet peaked at wavenumber k, its group velocity will be $\nu_g =$

1. $\frac{\hbar k}{2m} - u$
2. $\frac{\hbar k}{m} + u$
3. $\frac{\hbar k}{m} - u$
4. $-\frac{\hbar k}{2m} + u$

Q.No:7 TIFR-2020

In a one-dimensional system, the boundary condition that the derivative of the wavefunction $\psi'(x)$ should be continuous at every point is applicable whenever

1. the wavefunction $\psi(x)$ is itself continuous everywhere.
2. there is a bound state and the potential is piecewise continuous.
3. there is a bound state and the potential has no singularity anywhere.
4. there are bound or scattering states with definite momentum.

Q.No:8 TIFR-2020

A particle moving in one dimension, is placed in an asymmetric square well potential $V(x)$ as sketched below.

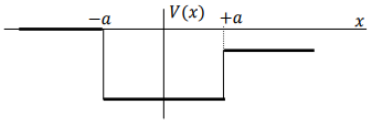


Figure 13: Asymmetric square well potential

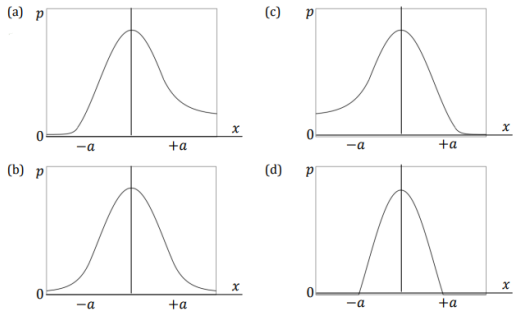


Figure 14: Probability distribution options

Q.No:9 TIFR-2022

A particle moves in one dimension x under the influence of a potential $V(x)$ as sketched in the figure below. The shaded region corresponds to infinite V, i.e., the particle is not allowed to penetrate there.

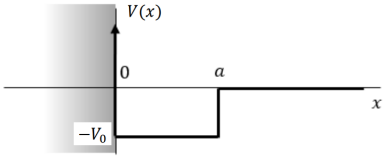


Figure 15: Potential well

If there is an energy eigenvalue $E = 0$, then a and V_0 are related by

1. $a^2V_0 = \frac{\left(n+\frac{1}{2}\right)^2\pi^2}{2m}$
2. $a^2V_0 = \frac{n^2\pi^2}{2m}$
3. $a^2V_0 = \frac{\left(n+\frac{1}{2}\right)\pi^2}{2m}$
4. $a^2V_0 = \frac{n\pi^2}{2m}$

Q.No:10 TIFR-2023

The value of the first derivative of the function

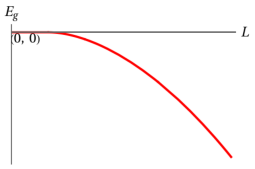
$$f(x) = \frac{2}{\sqrt{3}}e^{-\sqrt{3}x^2|x|}$$

at $x = 0$ is $f'(0) =$

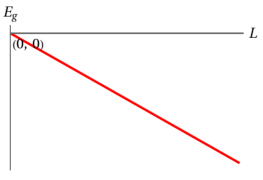
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1. Undefined
2. 0
3. 2
4. $2/\sqrt{3}$

(c)

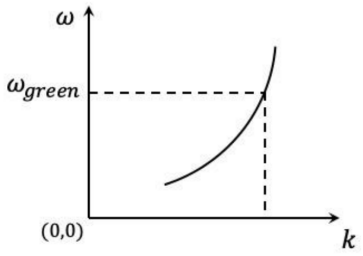


(d)



Q.No:11 TIFR-2026

For a medium, the dispersion relation for the propagation of light is shown below on a linear scale. Here, ω_{green} is the frequency of the green light. Which of the following statements is correct?



(a) The phase velocity for the green light is smaller than the group velocity at that frequency.

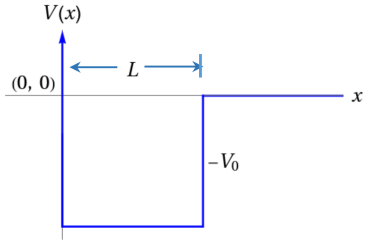
(b) The refractive index of the medium for the red light is smaller than that for the blue light.

(c) The phase velocity of the green light is larger than the group velocity at that frequency.

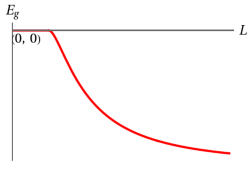
(d) The speed of light in this medium for green light is the same as that in vacuum.

Q.No:12 TIFR-2026

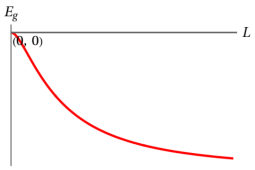
Consider a quantum particle in a one-dimensional potential of the shape shown, with the well of width L, and $V(x < 0) \to \infty$. The potential in the well is $-V_0$, and the potential for $x \in (L, \infty)$ is 0. Which plot describes the change in the ground state energy (E_g) as L is changed, keeping V_0 constant?



(a)



(b)



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Question No.	Entrance with year	Answer
Q.No:1	JEST-2012	Option d
Q.No:2	JEST-2014	Option b
Q.No:3	JEST-2020	Option c
Q.No:4	JEST-2021	Option d
Q.No:5	JEST-2022	Option A
Q.No:7	JEST-2023	Option 1
Q.No:8	JEST-2023	Option 2
Q.No:9	JEST-2024	Option 1
Q.No:10	JEST-2025	2
Q.No:1	TIFR-2012	Option c
Q.No:2	TIFR-2014	Option a
Q.No:3	TIFR-2015	Option b
Q.No:4	TIFR-2016	Option a
Q.No:5	TIFR-2017	Option c
Q.No:6	TIFR-2019	Option c
Q.No:7	TIFR-2020	Option c
Q.No:8	TIFR-2020	Option a
Q.No:9	TIFR-2022	Option a
Q.No:10	TIFR-2023	Option b
Q.No:11	TIFR-2026	Option a
Q.No:12	TIFR-2026	Option a

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