Identical Particles GATE, CSIR, JEST TIFR

https://snboseplc.com/identical-particles-gate-csir-jest-tifr/

GATE Questions

1 GATE-2012

Consider the wavefunction $\Psi = \psi(\vec{r}_1, \vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by,

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2) + \phi_2(\vec{r}_1)\phi_1(\vec{r}_2)]$$

where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha(+1/2)$ and $\beta(-1/2)$ should be

A. $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$

B. $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$

C. $\alpha\alpha$

D. $\beta\beta$

2 GATE-2014

The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

A. $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)]$

B. $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$

C. $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$

D. $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

3 GATE-2015

The Pauli matrices for three spin-1/2 particles are $\vec{\sigma}_1, \vec{\sigma}_2$, and $\vec{\sigma}_3$, respectively. The dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is _____.

4 GATE-2015

Consider a system of eight non-interacting, identical quantum particles of spin-3/2 in a one dimensional box of length L. The minimum excitation energy of the system, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is _____.

5 GATE-2016

A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0K$. The mass of the electron is m. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is _____.

6 GATE-2019

For a spin $\frac{1}{2}$ particle, let $|\uparrow\rangle$ and $|\downarrow\rangle$ denote its spin up and spin down states, respectively. If $|a\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$ and $|b\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$ are composite states of two such particles, which of the following statements is true for their total spin S?

A. $S = 1$ for $|a\rangle$, and $|b\rangle$ is not an eigenstate of the operator $\hat{S}^2$

B. $|a\rangle$ is not an eigenstate of the operator $\hat{S}^2$, and $S = 0$ for $|b\rangle$

C. $S = 0$ for $|a\rangle$, and $S = 1$ for $|b\rangle$

D. $S = 1$ for $|a\rangle$, and $S = 0$ for $|b\rangle$

7 GATE-2021

Three non-interacting bosonic particles of mass m each, are in a one-dimensional infinite potential well of width a. The energy of the third excited state of the system is $x \times \frac{\hbar^2\pi^2}{ma^2}$. The value of x (in integer) is _____.

8 GATE-2025

A system of five identical, non-interacting particles with mass m and spin $\frac{3}{2}$ is confined to a one-dimensional potential well of length L. If the lowest energy of the system is

$$N\frac{\pi^2\hbar^2}{2mL^2},$$

the value of N (in integer) is _____.

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1 CSIR Dec-2018

Three identical spin-$\frac{1}{2}$ particles of mass m are confined to a one-dimensional box of length L, but are otherwise free. Assuming that they are non-interacting, the energies of the lowest two energy eigenstates, in units of $\frac{\pi^2\hbar^2}{2mL^2}$, are

A. 3 and 6

B. 6 and 9

C. 6 and 11

D. 3 and 9

2 CSIR Dec-2019

Two spin $\frac{1}{2}$ fermions of mass m are confined to move in a one-dimensional infinite potential well of width L. If the particles are known to be in a spin triplet state, the ground state energy of the system (in units of $\frac{\hbar^2\pi^2}{2mL^2}$) is

A. 8

B. 2

C. 3

D. 5

3 Assam CSIR Dec-2019

The number density of a mono-atomic gas is $10^{18}m^{-3}$. The temperature at which the thermal de Broglie wavelength of the atoms (which have mass m) equals the average inter-atomic separation, is

A. $\frac{h^2}{2\pi mk_B} \times 10^{12}K$

B. $\frac{h^2}{2\pi mk_B} \times 10^{10}K$

C. $\frac{h^2}{2\pi mk_B} \times 10^{9}K$

D. $\frac{h^2}{2\pi mk_B} \times 10^{11}K$

4 Assam CSIR Dec-2019

The coordinates of two identical particles of spin 0 in one dimension are x_1 and x_2, respectively. Which of the following functions is an admissible wavefunction for the two-particle system?

A. $\frac{1}{2\pi a}e^{-\frac{1}{2a^2}(x_1^2+\frac{x_2^2}{4})}$

B. $\frac{1}{\pi a}e^{i(k_1x_1+k_2x_2)}e^{-\frac{1}{2a^2}(x_1^2+x_2^2)}$

C. $\frac{1}{2\pi a}e^{i(k_1x_1+k_2x_2)}e^{-\frac{1}{2a^2}(x_1^2+\frac{x_2^2}{4})}$

D. $\frac{1}{21\pi a^5}(x_1-x_2)^2e^{-\frac{1}{2a^2}(x_1^2+x_2^2)}$

5 Assam CSIR Dec-2019

Two distinguishable non-interacting particles, each of mass m are in a one-dimensional infinite square well in the interval $[0, a]$. If x_1 and x_2 are position operators of the two particles, the expectation value $\langle x_1x_2\rangle$ in the state in which one particle is in the ground state and the other one is in the first excited state, is

A. $\frac{1}{2}a^2$

B. $\frac{1}{2}\pi^2a^2$

C. $\frac{1}{4}a^2$

D. $\frac{1}{4}\pi^2a^2$

6 CSIR June -2023

A one-dimensional box contains one spin 0 particle and four spin $\frac{1}{2}$ particles. All particles have the same mass, hence the single particle energies are $E_n = n^2E$, $n = 1, 2, 3, \cdots$. The energies of the ground and the first excited states of the system of particles are, respectively,

A. $10E$ and $12E$

B. $11E$ and $20E$

C. $11E$ and $14E$

D. $10E$ and $13E$

7 CSIR Dec-2024

Two non-interacting identical spin-$\frac{1}{2}$ particles, each of mass m, are placed in a two-dimensional infinite square well of side L. The single-particle spatial wavefunction is

$$\varphi_{n_x,n_y}(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$$

where n_x and n_y are positive integers. If the particles are in a total spin state $|j = 1, m = 0\rangle$, the lowest possible energy eigenvalue is

A. $\frac{5\hbar^2\pi^2}{2mL^2}$

B. $\frac{\hbar^2\pi^2}{mL^2}$

C. $\frac{2\hbar^2\pi^2}{mL^2}$

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D. $\frac{7\hbar^2\pi^2}{2mL^2}$

8 CSIR June-2025

A system consists of two non-interacting identical spin-$\frac{1}{2}$ particles. The spatial wave functions for the individual particles are given by $\varphi_1(x)$ and $\varphi_2(x)$. Let x_1 and x_2 denote the positions of the particles respectively. The total wave function of the system (not necessarily normalized) can be

A. $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)]\,(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)$

B. $[\varphi_1(x_1)\varphi_1(x_2) + \varphi_2(x_1)\varphi_2(x_2)]\,|\uparrow\rangle_1|\uparrow\rangle_2$

C. $\varphi_1(x_1)\varphi_2(x_2)|\uparrow\rangle_1|\uparrow\rangle_2$

D. $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)]\,[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2]$

1 JEST-2012

The ground state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency ω is

A. $(15/2)\hbar\omega$

B. $(13/2)\hbar\omega$

C. $(1/2)\hbar\omega$

D. $5\hbar\omega$

2 JEST-2012

The spatial part of a two-electron state is symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is

A. $|\uparrow\rangle|\uparrow\rangle$

B. $|\uparrow\rangle|\downarrow\rangle$

C. $(|\downarrow\rangle|\uparrow\rangle - |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$

D. $(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$

3 JEST-2013

Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$. What would be the ground state wave function $\psi(x_1, x_2)$ if both electrons are arranged to have the same spin state?

A. $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) + \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)]$

B. $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)]$

C. $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

D. $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

4 JEST-2014

The lowest quantum mechanical energy of a particle confined in a one-dimensional box of size L is $2eV$. The energy of the quantum mechanical ground state for a system of three non-interacting spin $\frac{1}{2}$ particles is

A. $6eV$

B. $10eV$

C. $12eV$

D. $16eV$

5 JEST-2019

Consider a system of 15 non-interacting spin-polarized electrons. They are trapped in a two dimensional isotropic harmonic oscillator potential $V(x,y) = \frac{1}{2}m\omega^2(x^2+y^2)$. The angular frequency ω is such that $\hbar\omega = 1$ in some chosen unit. What is the ground state energy of the system in the same units?

6 JEST-2021

A one-dimensional box contains three identical particles in the ground state of the system. Find the ratio of total energies of these particles if they were spin-1/2 fermions, to that if they were bosons.

A. 1

B. 14/3

C. 2

D. 1/3

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7 JEST-2022

Consider eight electrons confined in a 1D box of length d. What is the minimum total energy for the system allowed by Pauli's exclusion principle?

A. $\frac{15h^2}{2md^2}$

B. $\frac{15h^2}{4md^2}$

C. $\frac{30h^2}{md^2}$

D. $\frac{15h^2}{8md^2}$

8 JEST-2022

Consider 5 identical spin 1/2 particles moving in a 3-dimensional harmonic oscillator potential,

$$V(r) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$$

The degeneracy of the ground state of the system is

A. 20

B. 7

C. 5

D. 32

9 JEST-2023

The spatial part of a two-electron state is anti-symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and the spin-down states respectively of each electron, the spin part of the two-electron state cannot be: (Q 21 has been cancelled due to typographical error.)

A. $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + (|\uparrow\rangle|\downarrow\rangle))$

B. $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - (|\uparrow\rangle|\downarrow\rangle))$

C. $|\uparrow\rangle|\downarrow\rangle$

D. $|\downarrow\rangle|\uparrow\rangle$

1 TIFR-2020

A quantum mechanical system consists of a one-dimensional infinite box, as indicated in the figures below.

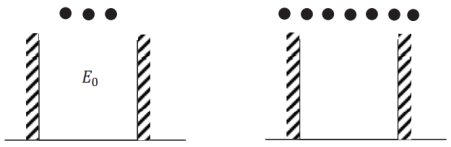


Figure 1: Figure for TIFR-2020 Q1. Source: snboseplc.com

3 (three) identical non-interacting spin-$\frac{1}{2}$ particles, are first placed in the box, and the ground state energy of the system is found to be $E_0 = 18$ eV. If 7 (seven) such identical particles are placed in the box, what will be the ground state energy, in units of eV?

2 TIFR-2020

Three noninteracting particles whose masses are in the ratio $1 : 4 : 16$ are placed together in the same harmonic oscillator potential $V(x)$.

The degeneracies of the first three energy eigenstates (ordered by increasing energy) will be

A. $1, 1, 1$

B. $1, 1, 2$

C. $1, 2, 1$

D. $1, 2, 2$

3 TIFR-2022

A system was formed of three spin-$\frac{1}{2}$ particles A, B and C, respectively and it was prepared in an initial state

$$|\psi\rangle = c_1|\uparrow\uparrow\uparrow\rangle + c_2|\uparrow\uparrow\downarrow\rangle + c_3|\uparrow\downarrow\uparrow\rangle + c_4|\uparrow\downarrow\downarrow\rangle$$

$$+c_5|\downarrow\uparrow\uparrow\rangle + c_6|\downarrow\uparrow\downarrow\rangle + c_7|\downarrow\downarrow\uparrow\rangle + c_8|\downarrow\downarrow\downarrow\rangle$$

where the symbols $|\uparrow\rangle$ and $|\downarrow\rangle$ indicate states with $S_z = +1/2$ (spin-up) and $S_z = -1/2$ (spin-down) respectively.

A measurement was made on the system in the initial state and this identified the spin state of the particle A to be $|\downarrow\rangle$ (spin-down). Now the expectation value of $\langle S_z\rangle$ for the particle C could be calculated as

A. $\frac{|c_5|^2+|c_7|^2-|c_6|^2-|c_8|^2}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

B. $\frac{c_5+c_7-c_6-c_8}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

C. $\frac{(|c_5|^*+|c_7|^*-|c_6|^*-|c_8|^*)(c_5+c_7-c_6-c_8)}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

D. $\frac{(c_5+c_7)^*(c_5+c_7)-(c_6+c_8)^*(c_6+c_8)}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

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Question No.	Entrance with year	
1	GATE-2012	Option B
2	GATE-2014	Option D
3	GATE-2015	8
4	GATE-2015	5
5	GATE-2016	24
6	GATE-2019	Option D
7	GATE-2021	5-6
8	GATE-2025	8
1	CSIR Dec-2018	Option 2
2	CSIR Dec-2019	Option 4
3	Assam CSIR Dec-2019	Option 4
4	Assam CSIR Dec-2019	Option 4
5	Assam CSIR Dec-2019	Option 3
6	CSIR June-2023	Option 3
7	CSIR Dec-2024	Option 4
8	CSIR June-2025	Option 1
1	JEST-2012	Option b
2	JEST-2012	Option c
3	JEST-2013	Option b
4	JEST-2014	Option c
5	JEST-2019	55
6	JEST-2021	Option c
7	JEST-2022	Option a
8	JEST-2022	Option a
9	JEST-2023	Option c
1	TIFR-2020	132
2	TIFR-2020	Option b
3	TIFR-2022	Option a

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