Linear Algebra and Formalism CSIR

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CSIR Questions

Q.No:1 CSIR-Dec-2014

The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

1. $b = 0$ or $a = 0$
2. $b = a$ or $b = -2a$
3. $b = 2a$ or $b = -a$
4. $b = a/2$ or $b = -a/2$

Q.No:2 CSIR-Dec-2014

Suppose the Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is

1. $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$
2. $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$
3. $-i\hbar\omega x\frac{\partial}{\partial x}$
4. $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

Q.No:3 CSIR-Dec-2014

Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x, p] = i$ in natural units ($\hbar = 1$). Then the commutator $[x, pe^{-p}]$ is

1. $i(1-p)e^{-p}$
2. $i(1-p^2)e^{-p}$
3. $i(1-e^{-p})$
4. ipe^{-p}

Q.No:4 CSIR-Dec-2014

Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication?

1. $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$
2. $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$, where $\alpha\beta \neq 1$
3. $\begin{pmatrix} \alpha & \alpha^* \\ \beta & \beta^* \end{pmatrix}$, where $\alpha\beta^*$ is real
4. $\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2+|\beta|^2= 1$

Q.No:5 CSIR-June-2015

The wavefunction of a particle in one dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p) = \int \psi(x)e^{-ipx/\hbar}dx$ in the momentum representation. If the action of an operator $\widehat{T}$ on $\psi(x)$ is given by $\widehat{T}\psi(x) = \psi(x + a)$, where a is a constant, then $\widehat{T}\phi(p)$ is given by

1. $-\frac{i}{\hbar}ap\phi(p)$
2. $e^{-iap/\hbar}\phi(p)$
3. $e^{+iap/\hbar}\phi(p)$
4. $\left(1 + \frac{i}{\hbar}ap\right)\phi(p)$

Q.No:6 CSIR-June-2015

A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where k(t) is a time dependent parameter. Then $\frac{d}{dt}\langle V\rangle$, the rate of change of the expectation value $\langle V\rangle$ of the potential energy, is

1. $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp + px\rangle$
2. $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{1}{2m}\langle p^2\rangle$
3. $\frac{k}{2m}\langle xp + px\rangle$
4. $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle$

Q.No:7 CSIR-June-2015

A particle of mass m is in a potential $V = \frac{1}{2}m\omega^2x^2$, where ω is a constant. Let $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right)$. In the Heisenberg picture $\frac{d\hat{a}}{dt}$ is given by

1. $\omega\hat{a}$
2. $-i\omega\hat{a}$
3. $\omega\hat{a}^\dagger$
4. $i\omega\hat{a}^\dagger$

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Q.No:8 CSIR-June-2015

Two different sets of orthogonal basis

$$\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\} \quad \text{and} \quad \left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$$

are given for a two-dimensional real vector space. The matrix representation of a linear operator A in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

1. $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$
2. $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
3. $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$
4. $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$

Q.No:9 CSIR-Dec-2016

The matrix $M = \begin{pmatrix}1 & 3 & 2\\3 & -1 & 0\\0 & 0 & 1\end{pmatrix}$ satisfies the equation

1. $M^3 - M^2 - 10M + 12I = 0$
2. $M^3 + M^2 - 12M + 10I = 0$
3. $M^3 - M^2 - 10M + 10I = 0$
4. $M^3 + M^2 - 10M + 10I = 0$

Q.No:10 CSIR-Dec-2016

Consider the operator $a = x + \frac{d}{dx}$ acting on smooth functions of x. The commutator $[a, \cos x]$ is

1. $-\sin x$
2. $\cos x$
3. $-\cos x$
4. 0

Q.No:11 CSIR-Dec-2016

Let $a = \frac{1}{\sqrt{2}}(x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2}}(x - ip)$ be the lowering and raising operators of a simple harmonic oscillator in units where the mass, angular frequency and $\hbar$ have been set to unity. If $|0\rangle$ is the ground state of the oscillator and λ is a complex constant, the expectation value of $\langle\psi|x|\psi\rangle$ in the state $|\psi\rangle = \exp(\lambda a^\dagger - \lambda^* a)|0\rangle$, is

1. $|\lambda|$
2. $\sqrt{|\lambda|^2 + \frac{1}{|\lambda|^2}}$
3. $\frac{1}{\sqrt{2}i}(\lambda - \lambda^*)$
4. $\frac{1}{\sqrt{2}}(\lambda + \lambda^*)$

Q.No:12 CSIR-Dec-2016

The 2×2 identity matrix I and the Pauli matrices $\sigma^x, \sigma^y, \sigma^z$ do not form a group under matrix multiplication. The minimum number of 2×2 matrices, which includes these four matrices, and form a group (under matrix multiplication) is

1. 20
2. 8
3. 12
4. 16

Q.No:13 CSIR-June-2017

Which of the following **cannot** be the eigenvalues of a real 3×3 matrix?

1. $2i, 0, -2i$
2. $1, 1, 1$
3. $e^{i\theta}, e^{-i\theta}, 1$
4. $i, 1, 0$

Q.No:14 CSIR-June-2017

The two vectors $\begin{pmatrix}a\\0\end{pmatrix}$ and $\begin{pmatrix}b\\c\end{pmatrix}$ are orthonormal if

1. $a = \pm 1, b = \pm 1/\sqrt{2}, c = \pm 1/\sqrt{2}$
2. $a = \pm 1, b = \pm 1, c = 0$
3. $a = \pm 1, b = 0, c = \pm 1$
4. $a = \pm 1, b = \pm 1/2, c = 1/2$

Q.No:15 CSIR-June-2017

Which of the following sets of 3×3 matrices (in which a and b are real numbers) forms a group under matrix multiplication?

1. $\left\{\begin{pmatrix}1 & 0 & a\\0 & 1 & 0\\b & 0 & 1\end{pmatrix}; a, b \in \mathbb{R}\right\}$
2. $\left\{\begin{pmatrix}1 & a & 0\\0 & 1 & b\\0 & 0 & 1\end{pmatrix}; a, b \in \mathbb{R}\right\}$

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3. $\left\{ \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} ; a, b \in \mathbb{R} \right\}$

4. $\left\{ \begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ; a, b \in \mathbb{R} \right\}$

Q.No:16 CSIR-Dec-2017

Consider the matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The condition for existence of a non-trivial solution, and the corresponding normalised solution (upto a sign) is

1. $b = 2c$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$
2. $c = 2b$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, 1, -2)$
3. $c = b + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(2, -1, -1)$
4. $b = c + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$

Q.No:17 CSIR-Dec-2017

Let A be a non-singular 3×3 matrix, the columns of which are denoted by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, respectively. Similarly, $\vec{u}, \vec{v}$ and $\vec{w}$ denote the vectors that form the corresponding columns of $(A^T)^{-1}$. Which of the following is true?

1. $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 1$
2. $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 1, \vec{u} \cdot \vec{c} = 0$
3. $\vec{u} \cdot \vec{a} = 1, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 0$
4. $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 0$

Q.No:18 CSIR-Dec-2017

Let x denote the position operator and p the canonically conjugate momentum operator of a particle. The commutator $\left[\frac{1}{2m}p^2 + \beta x^2, \frac{1}{m}p^2 + \gamma x^2\right]$, where β and γ are constants, is zero if

1. $\gamma = \beta$
2. $\gamma = 2\beta$
3. $\gamma = \sqrt{2}\beta$
4. $2\gamma = \beta$

Q.No:19 CSIR-Dec-2017

The Hamiltonian of a two-level quantum system is $H = \frac{1}{2}\hbar\omega \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. A possible initial state in which the probability of the system being in that quantum state does not change with time, is

1. $\begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix}$
2. $\begin{pmatrix} \cos\frac{\pi}{8} \\ \sin\frac{\pi}{8} \end{pmatrix}$
3. $\begin{pmatrix} \cos\frac{\pi}{2} \\ \sin\frac{\pi}{2} \end{pmatrix}$
4. $\begin{pmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{pmatrix}$

Q.No:20 CSIR-June-2018

Consider the three vectors $\vec{v}_1 = 2\hat{i} + 3\hat{k}$, $\vec{v}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{v}_3 = 5\hat{i} + \hat{j} + \alpha\hat{k}$, where $\hat{i}, \hat{j}$ and $\hat{k}$ are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of α is

1. $\frac{31}{4}$
2. $\frac{23}{4}$
3. $\frac{27}{4}$
4. 0

Q.No:21 CSIR-June-2018

Which of the following statements is true for a 3×3 real orthogonal matrix with determinant $+1$?

1. the modulus of each of its eigenvalues need not be 1, but their product must be 1
2. at least one of its eigenvalues is $+1$
3. all of its eigenvalues must be real
4. none of its eigenvalues need be real

Q.No:22 CSIR-Dec-2018

One of the eigenvalues of the matrix e^A is e^a, where $A = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix}$. The product of the other two eigenvalues of e^A is

1. e^{2a}
2. e^{-a}
3. e^{-2a}
4. 1

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Q.No:23 CSIR-Dec-2018

The product $\Delta x \Delta p$ of uncertainties in the position and momentum of a simple harmonic oscillator of mass m and angular frequency ω in the ground state $|0\rangle$, is $\hbar/2$. The value of the product $\Delta x \Delta p$ in the state $e^{-i\hat{p}\ell/\hbar}|0\rangle$ (where ℓ is a constant and $\hat{p}$ is the momentum operator) is

1. $\frac{\hbar}{2}\sqrt{\frac{m\omega\ell^2}{\hbar}}$
2. $\hbar$
3. $\frac{\hbar}{2}$
4. $\frac{\hbar^2}{m\omega\ell^2}$

Q.No:24 CSIR-Dec-2018

A 4×4 complex matrix A satisfies the relation $A^\dagger A = 4I$, where I is the 4×4 identity matrix. The number of independent real parameters of A is

1. 32
2. 10
3. 12
4. 16

Q.No:25 CSIR-June-2019

The elements of a 3×3 matrix A are the products of its row and column indices $A_{ij} = ij$ (where $i, j = 1, 2, 3$). The eigenvalues of A are

1. $(7, 7, 0)$
2. $(7, 4, 3)$
3. $(14, 0, 0)$
4. $(\frac{14}{3}, \frac{14}{3}, \frac{14}{3})$

Q.No:26 CSIR-June-2019

The operator A has a matrix representation $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ in the basis spanned by $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. In another basis spanned by $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, the matrix representation of A is

1. $\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$
2. $\begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}$
3. $\begin{pmatrix}3 & 1\\ 0 & 1\end{pmatrix}$
4. $\begin{pmatrix}3 & 0\\ 1 & 1\end{pmatrix}$

Q.No:27 CSIR-Dec-2019

If the rank of an $n \times n$ matrix A is m, where m and n are positive integers with $1 \leq m \leq n$, then the rank of the matrix A^2 is

1. m
2. $m - 1$
3. $2m$
4. $m - 2$

Q.No:28 CSIR-Dec-2019

Let the normalized eigenstates of the Hamiltonian $H = \begin{pmatrix}2 & 1 & 0\\ 1 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}$ be $|\psi_1\rangle, |\psi_2\rangle$ and $|\psi_3\rangle$. The expectation value $\langle H\rangle$ and the variance of H in the state $|\psi\rangle = \frac{1}{\sqrt{3}}(|\psi_1\rangle + |\psi_2\rangle - i|\psi_3\rangle)$ are

1. 4/3 and 1/3
2. 4/3 and 2/3
3. 2 and 2/3
4. 2 and 1

Q.No:29 CSIR-Dec-2019

Let $\hat{x}$ and $\hat{p}$ denote position and momentum operators obeying the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. If $|x\rangle$ denotes an eigenstate of $\hat{x}$ corresponding to the eigenvalue x, then $e^{ia\hat{p}/\hbar}|x\rangle$ is

1. an eigenstate of $\hat{x}$ corresponding to the eigenvalue x
2. an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x + a)$
3. an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x - a)$
4. not an eigenstate of $\hat{x}$

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Q.No:30 CSIR-Dec-2019

Consider the set of polynomials $\{x(t) = a_0 + a_1 t + \cdots + a_{n-1}t^{n-1}\}$ in t of degree less than n, such that $x(0) = 0$ and $x(1) = 1$. This set

1. constitutes a vector space of dimension n
2. constitutes a vector space of dimension $n-1$
3. constitutes a vector space of dimension $n-2$
4. does not constitute a vector space

Q.No:31 CSIR-Dec-2019

Consider the three vectors $\vec{A} = 3\hat{i}+2\hat{j}+\hat{k}$, $\vec{B} = 2\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{C} = -\hat{i} + y\hat{j} + 3\hat{k}$ in three-dimensional Euclidean space. The value of y for which $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar is

1. 1
2. 0
3. 2
4. -1

Q.No:32 CSIR-Dec-2019

The operator $A = \sum_{n=0}^{\infty} |n+1\rangle\langle n|$ is defined in terms of the eigenstates $|n\rangle$ of the number operator of the simple harmonic oscillator. Which of the following relations is obeyed by A and its hermitian conjugate $A^\dagger$? (In the following $\mathbf{1}$ is the identity operator.)

1. $A^\dagger A = \mathbf{1}$ and $AA^\dagger = \mathbf{1}$
2. $A^\dagger A = \mathbf{1}$, but $AA^\dagger \neq \mathbf{1}$
3. $A^\dagger A \neq \mathbf{1}$, but $AA^\dagger = \mathbf{1}$
4. $A^\dagger A \neq \mathbf{1}$ and $AA^\dagger \neq \mathbf{1}$

Q.No:33 CSIR-June-2020

The eigenvalues of the 3×3 matrix $M = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$ are

1. $a^2+b^2+c^2, 0, 0$
2. $b^2+c^2, a^2, 0$
3. $a^2+b^2, c^2, 0$
4. $a^2+c^2, b^2, 0$

Q.No:34 CSIR-Feb-2022

A two-state system evolves under the action of the Hamiltonian $H = E_0|A\rangle\langle A| + (E_0+\Delta)|B\rangle\langle B|$, where $|A\rangle$ and $|B\rangle$ are its two orthonormal states. These states transform to one another under parity, i.e. $P|A\rangle = |B\rangle$ and $P|B\rangle = |A\rangle$. If at time $t = 0$ the system is in a state of definite parity $P = 1$, the earliest time t at which the probability of finding the system in a state of parity $P = -1$ is one is

1. $\frac{\pi\hbar}{2\Delta}$
2. $\frac{\pi\hbar}{\Delta}$
3. $\frac{3\pi\hbar}{2\Delta}$
4. $\frac{2\pi\hbar}{\Delta}$

Q.No:35 CSIR-Feb-2022

A generic 3×3 real matrix A has eigenvalues $0, 1$ and 6, and I is the 3×3 identity matrix. The quantity/quantities that cannot be determined from this information is/are the

1. eigenvalue of $(I+A)^{-1}$
2. eigenvalue of $(I+A^TA)$
3. eigenvalue of A^TA
4. rank of A

Q.No:36 CSIR-Sep-2022

Two $n\times n$ invertible real matrices A and B satisfy the relation

$$(AB)^T = -(A^{-1}B)^{-1}$$

If B is orthogonal then A must be

1. lower triangle
2. orthogonal
3. symmetric
4. antisymmetric

Q.No:37 CSIR-Sep-2022

In terms of a complete set of orthonormal basis kets $|n\rangle$, $n = 0, \pm 1, \pm 2, ...,$ the Hamiltonian is

$$H = \sum_n (E|n\rangle\langle n| + \epsilon|n+1\rangle\langle n| + \epsilon|n\rangle\langle n+1|)$$

where E and ϵ are constants. The state $|\varphi\rangle = \sum_n e^{in\varphi}|n\rangle$ is an eigenstate with energy

1. $E + \epsilon\cos\varphi$

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2. $E - \epsilon \cos\varphi$
3. $E + 2\epsilon \cos\varphi$
4. $E - 2\epsilon \cos\varphi$

Q.No:38 CSIR-Sep-2022

The matrix corresponds to differential operator $(1 + \frac{d}{dx})$ in the space of polynomial of degree at most two in the basis spanned by $f_1 = 1$, $f_2 = x$ and $f_3 = x^2$, is

1. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$
4. $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

Q.No:39 CSIR-June-2023

Two operators A and B satisfy the commutation relations $[H, A] = -\hbar\omega B$ and $[H, B] = \hbar\omega A$, where ω is a constant and H is the Hamiltonian of the system. The expectation value $\langle A\rangle_\psi(t) = \langle\psi|A|\psi\rangle$ in a state $|\psi\rangle$, such that at time $t = 0$, $\langle A\rangle_\psi(0) = 0$ and $\langle B\rangle_\psi(0) = i$, is

1. $\sin(\omega t)$
2. $\sinh(\omega t)$
3. $\cos(\omega t)$
4. $\cosh(\omega t)$

Q.No:40 CSIR-June-2024

If A and B are Hermitian operators and C is an antihermitian operator, then:

1. $[[A, B], C]$ is Hermitian and $[[A, C], B]$ is antihermitian
2. $[[A, B], C]$ and $[[A, C], B]$ are both antihermitian
3. $[[A, B], C]$ and $[[A, C], B]$ are both Hermitian
4. $[[A, B], C]$ is antihermitian and $[[A, C], B]$ is Hermitian

Q.No:41 CSIR-June-2024

The matrix A is given by

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{pmatrix}$$

The eigenvalues of $3A^3 + 5A^2 - 6A + 2I$, where I is the identity matrix, are

1. $4, 9, 27$
2. $1, 9, 44$
3. $1, 110, 8$
4. $4, 110, 10$

Q.No:42 CSIR-June-2023

The matrix $M = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ satisfies the equation $M^3 + \alpha M^2 + \beta M + 3 = 0$ if (α, β) are

1. $(-2, 2)$
2. $(-3, 3)$
3. $(-6, 6)$
4. $(-4, 4)$

Q.No:43 CSIR-June-2023

The matrix $R_{\hat{\mathbf{n}}}(\theta)$ represents a rotation by an angle θ about the axis $\hat{\mathbf{n}}$. The value of θ and $\hat{\mathbf{n}}$ corresponding to the matrix $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}$, respectively, are

1. $\pi/2$ and $(0, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}})$
2. $\pi/2$ and $(0, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}})$
3. π and $(0, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}})$
4. π and $(0, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}})$

Q.No:44 CSIR-Dec-2023

Let $\mathbf{M}$ be a 3×3 real matrix such that

$$e^{\mathbf{M}\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where θ is a real parameter. Then $\mathbf{M}$ is given by

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1. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
3. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q.No:45 CSIR-Dec-2024

A particle of mass m is in a cubic box of side a. The potential inside the box ($0 \le x \le a,\ 0 \le y \le a,\ 0 \le z \le a$) is zero and infinite outside. If the particle is in an energy eigenstate with $E = \frac{7\pi^2\hbar^2}{ma^2}$, a possible wavefunction is

1. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{a}\right) \sin\left(\frac{2\pi z}{a}\right)$
2. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{3\pi y}{a}\right) \sin\left(\frac{\pi z}{a}\right)$
3. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi y}{a}\right) \sin\left(\frac{3\pi z}{a}\right)$
4. $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi y}{a}\right) \sin\left(\frac{2\pi z}{a}\right)$

Q.No:46 CSIR-Dec-2024

The constant B which makes e^{-ax^2} an eigenfunction of the operator $\left(\frac{d^2}{dx^2} - Bx^2\right)$ is

1. $4a^2$
2. 0
3. $2a$
4. 1

Q.No:47 CSIR-Dec-2024

If I is an $n \times n$ identity matrix and

$$adj(2I) = 2^k I,$$

then k is equal to

1. 1
2. n
3. $n-1$
4. 2

Q.No:48 CSIR-June-2025

For the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which of the following is true?

1. $A^3 = 5A^2 - 4A - 2I$
2. $A^3 = 4A^2 - 6A + 3I$
3. $A^3 = 5A^2 - 5A - I$
4. $A^3 = 8A^2 + 3A - 4I$

Q.No:49 CSIR-June-2025

The Hamiltonian of the 1-dimensional quantum harmonic oscillator is given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. The expectation value of $[D, H]$ in the ground state, where $D = \frac{1}{2\hbar}(xp + px)$, is (in units of $\hbar\omega$):

1. i
2. $\frac{1}{2}$
3. $-\frac{3i}{2}$
4. 0

Q.No:50 CSIR-Dec-2025

The commutator of two matrices A and B is defined as $[A, B] = AB - BA$ and the anti-commutator as $\{A, B\} = AB + BA$. If $\{A, B\} = 0$, then we can express $[A, BC]$ as

1. $B\{A, C\}$
2. $-B[A, C]$
3. $-B\{A, C\}$
4. $[A, B]C$

Q.No:51 CSIR-June-2026

In a 4-dimensional vector space V_4, three linearly independent vectors are given by $e_1 = (1, 1, 0, 0)$, $e_2 = (0, 1, 1, 0)$, $e_3 = (0, 0, 1, 1)$.

Which of the following vector e_4 will make $\{e_1, e_2, e_3, e_4\}$ a basis of V_4?

1. $(1, 1, 1, 1)$
2. $(1, 1, 1, 0)$
3. $(1, 0, -1, 0)$
4. $(0, 1, 1, 0)$

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Q.No:52 CSIR-June-2026

You are given a time-independent observable O for a system with time-independent Hamiltonian H and a general time-dependent state $|\phi(t)\rangle$. If the expectation value is $\langle O(t)\rangle_\phi = \langle\phi(t)|O|\phi(t)\rangle$, then consider the following statements (A) to (D) and choose the correct option.

(A) If O commutes with H, then $\langle O(t)\rangle_\phi$ is independent of time.

(B) If $|\phi(t)\rangle$ is an eigenstate of H, then $\langle O(t)\rangle_\phi$ is independent of time.

(C) If $|\phi(t)\rangle$ is an eigenstate of O, then $\langle O(t)\rangle_\phi$ is independent of time.

(D) If $|\phi(t)\rangle$ is an eigenstate of $[O, H]$, then $\langle O(t)\rangle_\phi$ is independent of time.

1. Only (A) and (B) are correct.
2. Only (A) is correct.
3. Only (B), (C) and (D) are correct.
4. Only (A), (B) and (C) are correct.

Q.No:53 CSIR-June-2026

A three-state system is described by a certain Hamiltonian having eigenenergies E_0, $2E_0$, and $3E_0$, with the corresponding eigenstates

$$\begin{pmatrix}1\\1\\1\end{pmatrix}, \quad \begin{pmatrix}1\\0\\-1\end{pmatrix}, \quad \begin{pmatrix}1\\-2\\1\end{pmatrix},$$

respectively. If the state of the system at $t = 0$ is

$$\begin{pmatrix}1\\0\\0\end{pmatrix},$$

then what is the probability of finding the system in the same state at $t = \frac{\pi\hbar}{2E_0}$?

1. 1
2. $\frac{1}{3}$
3. $\frac{5}{18}$
4. $\frac{2}{9}$

Q.No:54 CSIR-June-2026

Consider two operators $\hat{R}$ and $\hat{S}$, where $\hat{R}$ commutes with $[\hat{R}, \hat{S}]$. If α is a constant, then $\left[e^{\alpha\hat{R}}, \hat{S}\right]$ is:

1. $\left[\hat{R}^\alpha, \hat{S}\right]$
2. $\alpha e^{\alpha\hat{R}}[\hat{R}, \hat{S}]$
3. 0
4. $\alpha e^{\hat{R}}[e^\alpha\hat{R}, \hat{S}]$

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Question No.	Entrance with year	Answer
1	CSIR-Dec-2014	Option 2
2	CSIR-Dec-2014	Option 2
3	CSIR-Dec-2014	Option 1
4	CSIR-Dec-2014	Option 4
5	CSIR-June-2015	Option 3
6	CSIR-June-2015	Option 1
7	CSIR-June-2015	Option 2
8	CSIR-June-2015	Option 3
9	CSIR-Dec-2016	Option 3
10	CSIR-Dec-2016	Option 1
11	CSIR-Dec-2016	Option 4
12	CSIR-Dec-2016	Option 4
13	CSIR-June-2017	Option 4
14	CSIR-June-2017	Option 3
15	CSIR-June-2017	Option 3
16	CSIR-Dec-2017	Option 4
17	CSIR-Dec-2017	Option 3
18	CSIR-Dec-2017	Option 2
19	CSIR-Dec-2017	Option 2
20	CSIR-June-2018	Option 1
21	CSIR-June-2018	Option 2
22	CSIR-Dec-2018	Option 4
23	CSIR-Dec-2018	Option 3
24	CSIR-Dec-2018	Option 4
25	CSIR-June-2019	Option 3
26	CSIR-June-2019	Option 2
27	CSIR-Dec-2019	Option 1
28	CSIR-Dec-2019	Option 3
29	CSIR-Dec-2019	Option 3
30	CSIR-Dec-2019	Option 4
31	CSIR-Dec-2019	Option 2
32	CSIR-Dec-2019	Option 2
33	CSIR-June-2020	Option 1
34	CSIR-Feb-2022	Option 2
35	CSIR-Feb-2022	Option 2
36	CSIR-Sep-2022	Option 4
37	CSIR-Sep-2022	Option 3
38	CSIR-Sep-2022	Option 1
39	CSIR-June-2023	Option 2
40	CSIR-June-2024	Option 2
41	CSIR-June-2024	Option 4
42	CSIR-June-2023	Option 3
43	CSIR-June-2023	Option 4
44	CSIR-Dec-2023	Option 2
45	CSIR-Dec-2024	Option 3
46	CSIR-Dec-2024	Option 1
47	CSIR-Dec-2024	Option 3
48	CSIR-June-2025	Option 1
49	CSIR-June-2025	Option 4
50	CSIR-Dec-2025	Option 3
51	CSIR-June-2026	Option 2
52	CSIR-June-2026	Option 4
53	CSIR-June-2026	Option 3
54	CSIR-June-2026	Option 2

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