# Linear Algebra and Formalism GATE

**https://snboseplc.com/b-qm3-linear-algebra-and-formalism-gate/**

## Q.No:1 GATE-2012

Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$

1. The vectors $\vec{a}$ and $\vec{b}$ are linearly independent
2. The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent
3. The vectors $\vec{a}$ and $\vec{b}$ are orthogonal
4. The vectors $\vec{a}$ and $\vec{b}$ are normalized

## Q.No:2 GATE-2012

The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

1. $0, 1, 1$
2. $0, -\sqrt{2}, \sqrt{2}$
3. $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$
4. $\sqrt{2}, \sqrt{2}, 0$

## Q.No:3 GATE-2013

The degenerate eigenvalue of the matrix $\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$ is (your answer should be an integer) _____

## Q.No:4 GATE-2014

The matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

1. orthogonal
2. symmetric
3. anti-symmetric
4. unitary

## Q.No:5 GATE-2014

$\psi_1$ and $\psi_2$ are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state $\psi_2$, its probability to be in the spin-up state is _____.

## Q.No:6 GATE-2016

Which of the following operators is Hermitian?

1. $\frac{d}{dx}$
2. $\frac{d^2}{dx^2}$
3. $i\frac{d^2}{dx^2}$
4. $\frac{d^3}{dx^3}$

## Q.No:7 GATE-2016

If $x$ and $p$ are the $x$ components of the position and the momentum operators of a particle respectively, the commutator $[x^2, p^2]$ is

1. $i\hbar(xp + px)$
2. $2i\hbar(xp - px)$
3. $i\hbar(xp + px)$
4. $2i\hbar(xp + px)$

## Q.No:8 GATE-2016

For the parity operator $P$, which of the following statements is NOT true?

1. $P^\dagger = P$
2. $P^2 = -P$
3. $P^2 = I$
4. $P^\dagger = P^{-1}$

## Q.No:9 GATE-2017

The Poisson bracket $[x, xp_y + yp_x]$ is equal to

1. $-x$
2. $y$
3. $2p_x$
4. $p_y$

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## Q.No:10 GATE-2017

Let $X$ be a column vector of dimension $n > 1$ with at least one non-zero entry. The number of non-zero eigenvalues of the matrix $M = XX^T$ is

1. 0
2. $n$
3. 1
4. $n - 1$

## Q.No:11 GATE-2017

Which one of the following operators is Hermitian?

1. $i\frac{(p_x x^2 - x^2 p_x)}{2}$
2. $i\frac{(p_x x^2 + x^2 p_x)}{2}$
3. $e^{ip_x a}$
4. $e^{-ip_x a}$

## Q.No:12 GATE-2018

The eigenvalues of a Hermitian matrix are all

1. real
2. imaginary
3. of modulus one
4. real and positive

## Q.No:13 GATE-2018

Given $\vec{V}_1 = \hat{i} - \hat{j}$ and $\vec{V}_2 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1, \vec{V}_2, \vec{V}_3)$ a complete set for a three dimensional real linear vector space?

1. $\vec{V}_3 = \hat{i} + \hat{j} + 4\hat{k}$
2. $\vec{V}_3 = 2\hat{i} - \hat{j} + 2\hat{k}$
3. $\vec{V}_3 = \hat{i} + 2\hat{j} + 6\hat{k}$
4. $\vec{V}_3 = 2\hat{i} + \hat{j} + 4\hat{k}$

## Q.No:14 GATE-2018

If $H$ is the Hamiltonian for a free particle with mass $m$, the commutator $[x, [x, H]]$ is

1. $\hbar^2/m$
2. $-\hbar^2/m$
3. $-\hbar^2/(2m)$
4. $\hbar^2/(2m)$

## Q.No:15 GATE-2019

During a rotation, vectors along the axis of rotation remain unchanged. For the rotation matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$, the unit vector along the axis of rotation is

1. $\frac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k})$
2. $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$
3. $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$
4. $\frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$

## Q.No:16 GATE-2019

Let $|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent two possible states of a two-level quantum system. The state obtained by the incoherent superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$ is given by a density matrix that is defined as $\rho \equiv c_1|\psi_1\rangle\langle\psi_1| + c_2|\psi_2\rangle\langle\psi_2|$. If $c_1 = 0.4$ and $c_2 = 0.6$, the matrix element $\rho_{22}$ (rounded off to one decimal place) is _____.

## Q.No:17 GATE-2019

The Hamiltonian operator for a two-level quantum system is $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$. If the state of the system at $t = 0$ is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ then $|\langle\psi(0)|\psi(t)\rangle|^2$ at a later time $t$ is

1. $\frac{1}{2}\left(1 + e^{-(E_1 - E_2)t/\hbar}\right)$
2. $\frac{1}{2}(1 - e^{-(E_1 - E_2)t/\hbar})$
3. $\frac{1}{2}(1 + \cos[(E_1 - E_2)t/\hbar])$
4. $\frac{1}{2}(1 - \cos[(E_1 - E_2)t/\hbar])$

## Q.No:18 GATE-2020

A real, invertible $3 \times 3$ matrix $M$ has eigenvalues $\lambda_i$, $(i = 1, 2, 3)$ and the corresponding eigenvectors are $|e_i\rangle$, $(i = 1, 2, 3)$ respectively. Which one of the following is correct?

1. $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1, 2, 3$
2. $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1, 2, 3$
3. $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$, for $i = 1, 2, 3$
4. The eigenvalues of $M$ and $M^{-1}$ are not related.

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## Q.No:19 GATE-2020

The product of eigenvalues of $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is

1. $-1$
2. 1
3. 0
4. 2

## Q.No:20 GATE-2020

Let $|e_1\rangle \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |e_2\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $|e_3\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let $S = \{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$. Let $\mathbb{R}^3$ denote the three-dimensional real vector space. Which one of the following is correct?

1. $S$ is an orthonormal set
2. $S$ is a linearly dependent set
3. $S$ is a basis for $\mathbb{R}^3$
4. $\sum_{i=1}^{3} |e_i\rangle\langle e_i| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

## Q.No:21 GATE-2020

Which one of the following matrices does NOT represent a proper rotation in a plane?

1. $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{pmatrix}$
2. $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
3. $\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$
4. $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

## Q.No:22 GATE-2021

$P$ and $Q$ are two Hermitian matrices and there exists a matrix $R$, which diagonalizes both of them, such that $RPR^{-1} = S_1$ and $RQR^{-1} = S_2$, where $S_1$ and $S_2$ are diagonal matrices. The correct statement(s) is(are)

1. All the elements of both matrices $S_1$ and $S_2$ are real.
2. The matrix $PQ$ can have complex eigenvalues.
3. The matrix $QP$ can have complex eigenvalues.
4. The matrices $P$ and $Q$ commute.

## Q.No:23 GATE-2022

An electromagnetic pulse has a pulse width of $10^{-3}$ s. The uncertainty in the momentum of the corresponding photon is of the order of $10^{-N}$ kg m s$^{-1}$, where $N$ is an integer. The value of $N$ is _____ (speed of light $= 3 \times 10^8$ m s$^{-1}$, $h = 6.6 \times 10^{-34}$ J s)

## Q.No:24 GATE-2022

What is the maximum number of free independent real parameters specifying an $n$-dimensional orthogonal matrix?

1. $n(n-2)$
2. $(n-1)^2$
3. $\frac{n(n-1)}{2}$
4. $\frac{n(n+1)}{2}$

## Q.No:25 GATE-2023

A $4 \times 4$ matrix $M$ has the property $M^\dagger = -M$ and $M^4 = 1$, where 1 is the $4 \times 4$ identity matrix. Which one of the following is the CORRECT set of eigenvalues of the matrix $M$?

1. $(1, 1, -1, -1)$
2. $(i, i, -i, -i)$
3. $(i, i, i, -i)$
4. $(1, 1, -i, -i)$

## Q.No:26 GATE-2023

Which of the following options represent(s) linearly independent pair(s) of functions of a real variable $x$?

1. $e^{ix}$ and $e^{-ix}$
2. $x$ and $e^x$
3. $2^x$ and $2^{-3+x}$
4. $e^{ix}$ and $\sin x$

## Q.No:27 GATE-2023

The wavefunction of a particle in one dimension is given by

$$\psi(x) = \begin{cases} M & -a < x < a \\ 0 & \text{otherwise.} \end{cases}$$

Here $M$ and $a$ are positive constants. If $\varphi(p)$ is the corresponding momentum space wavefunction, which one of the following plots best represents $|\varphi(p)|^2$ ?

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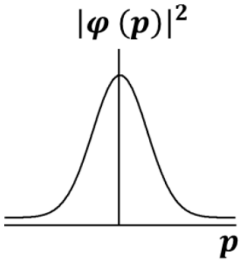



Figure 1: (A)

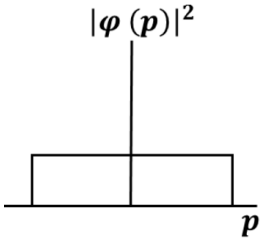



Figure 2: (B)

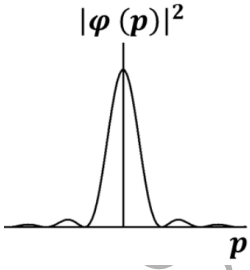



Figure 3: (C)

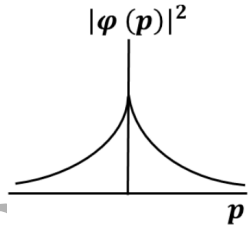



Figure 4: (D)

## Q.No:28 GATE-2024

Consider two matrices: $P = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Which of the following statement is/are true?

1. $P$ and $Q$ have the same set of eigenvalues
2. $P$ and $Q$ commute with each other
3. $P$ and $Q$ have different sets of linearly independent eigenvectors
4. $P$ is diagonalizable

## Q.No:29 GATE-2025

Consider the set $\{1, x, x^2\}$. An orthonormal basis in $x \in [-1, 1]$ is formed from these three terms, where the normalization of a function $f(x)$ is defined via

$$\int_{-1}^{1} x^2 \, [f(x)]^2 \, dx = 1$$

If the orthonormal basis set is

$$\left( \sqrt{\frac{3}{2}}, \; \sqrt{\frac{5}{2}}\, x, \; \frac{1}{2}\sqrt{\frac{21}{N}}\,(5x^2 - 3) \right)$$

then the value of $N$ (in integer) is _____

## Q.No:30 GATE-2026

Consider an operator $\hat{A}$ which is not Hermitian. Find the possible values of $c$ and $d$ such that the operator $(c\hat{A} - d\hat{A}^\dagger)$ is Hermitian.

(A) $c = i$ and $d = i$

(B) $c = 1$ and $d = 1$

(C) $c = -1$ and $d = i$

(D) $c = i$ and $d = -i$

## Q.No:31 GATE-2026

Which of the following operators is/are self-adjoint?

(A) $x^2 \frac{d^2}{dx^2} + 3x\frac{d}{dx} + x^2$

(B) $(1 - x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 3x$

(C) $(3x - 4x^3)\frac{d^2}{dx^2} + (3 - 12x^2)\frac{d}{dx} + 12$

(D) $x\frac{d^2}{dx^2} + x^2\frac{d}{dx} + \frac{5x}{3}$

## Q.No:32 GATE-2026

Consider two operators $\hat{A}$ and $\hat{B}$ which are related as $\hat{A} = \exp(i\theta\hat{B})$. If $\theta$ is a non-zero real number, which of the following statements is/are true?

(A) If $\hat{B}$ is Hermitian, then $\hat{A}$ is unitary.

(B) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is unitary.

(C) If $\hat{B}$ is Hermitian, then $\left|\mathrm{Det}(\hat{A})\right| = 1$.

(D) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is Hermitian.

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## Q.No:33 GATE-2026

Consider operators $\hat{A}$, $\hat{B}$, and $\hat{C}$ for three observables of a quantum system satisfying $[\hat{A}, \hat{B}] = 0$, $[\hat{B}, \hat{C}] = 0$, $[\hat{A}, \hat{C}] \neq 0$, with uncertainties $\Delta A$, $\Delta B$, and $\Delta C$, respectively. From the options given below, which is/are implied by the commutation relations among $\hat{A}$, $\hat{B}$, and $\hat{C}$?

(A) $\Delta A \, \Delta B > 0$

(B) $\Delta A \, \Delta C > 0$

(C) $\hat{A}$ and $\hat{B}$ can be simultaneously diagonalized.

(D) $\hat{A}$, $\hat{B}$, and $\hat{C}$ can be simultaneously diagonalized.

| Question No. | Entrance with year | Answer |
|---|---|---|
| 1 | GATE-2012 | Option A |
| 2 | GATE-2012 | Option B |
| 3 | GATE-2013 | 5 |
| 4 | GATE-2014 | Option D |
| 5 | GATE-2014 | 0.66 - 0.68 |
| 6 | GATE-2016 | Option B |
| 7 | GATE-2016 | Option D |
| 8 | GATE-2016 | Option B |
| 9 | GATE-2017 | Option B |
| 10 | GATE-2017 | Option C |
| 11 | GATE-2017 | Option A |
| 12 | GATE-2018 | Option A |
| 13 | GATE-2018 | Option D |
| 14 | GATE-2018 | Option B |
| 15 | GATE-2019 | Option B |
| 16 | GATE-2019 | 0.6 |
| 17 | GATE-2019 | Option C |
| 18 | GATE-2020 | Option B |
| 19 | GATE-2020 | Option A |
| 20 | GATE-2020 | Option C |
| 21 | GATE-2020 | Option D |
| 22 | GATE-2021 | Options A & D |
| 23 | GATE-2022 | 39 - 40 |
| 24 | GATE-2022 | Option c |
| 25 | GATE-2023 | Option B |
| 26 | GATE-2023 | Options A, B, D |
| 27 | GATE-2023 | Option C |
| 28 | GATE-2024 | Options A, B, C |
| 29 | GATE-2025 | 6 |
| 30 | GATE-2026 | Option A |
| 31 | GATE-2026 | Option B,C |
| 32 | GATE-2026 | Option A,C,D |
| 33 | GATE-2026 | Option B,C |

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