Linear Algebra and formalism TIFR

https://snboseplc.com/b-qm3-linear-algebra-and-formalism-tifr/

TIFR Questions

Q.No:1 TIFR-2012

Two different 2×2 matrices A and B are found to have the same eigenvalues. It is then correct to state that $A = SBS^{-1}$ where S can be a

1. traceless 2×2 matrix
2. Hermitian 2×2 matrix
3. unitary 2×2 matrix
4. arbitrary 2×2 matrix

Q.No:2 TIFR-2013

Consider a quantum mechanical system with three linear operators $\hat{A}, \hat{B}$ and $\hat{C}$, which are related by

$$\hat{A}\hat{B} - \hat{C} = \hat{I}$$

where $\hat{I}$ is the unit operator. If $\hat{A} = d/dx$ and $\hat{B} = x$, then $\hat{C}$ must be

1. zero
2. $\frac{d}{dx}$
3. $-x\frac{d}{dx}$
4. $x\frac{d}{dx}$

Q.No:3 TIFR-2013

The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be $+1$ is

1. $1/\sqrt{2}$
2. $1/2$
3. 1
4. $1/4$

Q.No:4 TIFR-2014

The product **MN** of two Hermitian matrices **M** and **N** is anti-Hermitian. It follows that

1. $\{\mathbf{M}, \mathbf{N}\} = 0$
2. $[\mathbf{M}, \mathbf{N}] = 0$
3. $\mathbf{M}^\dagger = \mathbf{N}$
4. $\mathbf{M}^\dagger = \mathbf{N}^{-1}$

Q.No:5 TIFR-2015

1000 neutral spinless particles are confined in a one-dimensional box of length $100nm$. At a given instant of time, if 100 of these particle have energy $4\epsilon_0$ and the remaining 900 have energy $225\epsilon_0$, then the number of particles in the left half of the box will be approximately

1. 625
2. 500
3. 441
4. 100

Q.No:6 TIFR-2016

If the eigenvalues of a symmetric 3×3 matrix **A** are $0, 1, 3$ and the corresponding eigenvectors can be written as

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

respectively, then the matrix $\mathbf{A}^4$ is

1. $\begin{pmatrix} 41 & -81 & 40 \\ -81 & 0 & -81 \\ 40 & -81 & 41 \end{pmatrix}$
2. $\begin{pmatrix} -82 & -81 & 79 \\ -81 & 81 & -81 \\ 79 & -81 & 83 \end{pmatrix}$
3. $\begin{pmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{pmatrix}$
4. $\begin{pmatrix} 14 & -13 & 27 \\ -13 & 54 & -13 \\ 27 & -13 & 14 \end{pmatrix}$

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Q.No:7 TIFR-2016

A particle moving in one dimension is confined inside a rigid box located between $x = -a/2$ and $x = a/2$. If the particle is in its ground state

$$\psi_0(x) = \sqrt{2/a}\cos\frac{\pi x}{a}$$

the quantum mechanical probability of its having a momentum p is given by

1. $\frac{8\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$
2. $\frac{\pi^2\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\sin^2\frac{pa}{2\hbar}$
3. $\frac{2\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$
4. $\frac{16\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}$

Q.No:8 TIFR-2017

Denote the commutator of two matrices A and B by $[A, B] = AB - BA$ and the anti-commutator by $\{A, B\} = AB + BA$.

If $\{A, B\} = 0$, we can write $[A, BC] =$

1. $-B[A, C]$
2. $B\{A, C\}$
3. $-B\{A, C\}$
4. $[A, C]B$

Q.No:9 TIFR-2017

The matrix

$$\begin{pmatrix} 100\sqrt{2} & x & 0 \\ -x & 0 & -x \\ 0 & x & 100\sqrt{2} \end{pmatrix}$$

where $x > 0$, is known to have two equal eigenvalues. Find the value of x.

Q.No:10 TIFR-2017

A unitary matrix U is expanded in terms of a Hermitian matrix H, such that

$$U = e^{i\pi H/2}$$

If we know that

$$H = \begin{pmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & -1/2 \end{pmatrix}$$

then U must be

1. $\begin{pmatrix} i & 1/2 & \sqrt{3}/2 \\ 1/2 & i & 1/2 \\ \sqrt{3}/2 & 1/2 & i \end{pmatrix}$
2. $\begin{pmatrix} i/2 & 0 & i\sqrt{3}/2 \\ 0 & i & 0 \\ i\sqrt{3}/2 & 0 & -i/2 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ \sqrt{3} & 0 & -1 \end{pmatrix}$
4. $\begin{pmatrix} 2i & 1 & \sqrt{3}/2 \\ 1 & 2i & 0 \\ \sqrt{3}/2 & 0 & 2i \end{pmatrix}$

Q.No:11 TIFR-2018

If a 2×2 matrix $\mathbb{M}$ is given by

$$\mathbb{M} = \begin{pmatrix} 1 & (1-i)/\sqrt{2} \\ (1+i)/\sqrt{2} & 0 \end{pmatrix}$$

then $\det \exp \mathbb{M} =$

1. e
2. e^2
3. $2i\sin\sqrt{2}$
4. $\exp(-2\sqrt{2})$

Q.No:12 TIFR-2019

The eigenvalues of a 3×3 matrix $\mathbb{M}$ are

$$\lambda_1 = 2 \; \lambda_2 = -1 \; \lambda_3 = 1$$

and the eigenvectors are

$$e_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} e_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

The matrix $\mathbb{M}$ is

1. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$
2. $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$
4. $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

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Q.No:13 TIFR-2020

The eigenvector e_1 corresponding to the smallest eigenvalue of the matrix

$$\begin{pmatrix} 2a^2 & a & 0 \\ a & 1 & a \\ 0 & a & 2a^2 \end{pmatrix}$$

where $a = \sqrt{\frac{3}{2}}$, is given (in terms of its transpose) by

1. $e_1^T = \frac{1}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\sqrt{3} & \frac{1}{\sqrt{2}} \end{pmatrix}$
2. $e_1^T = \frac{1}{2}\begin{pmatrix} \sqrt{\frac{3}{2}} & 1 & \sqrt{\frac{3}{2}} \end{pmatrix}$
3. $e_1^T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$
4. $e_1^T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$

Q.No:14 TIFR-2020

The momentum operator

$$i\hbar\frac{d}{dx}$$

acts on a wavefunction $\psi(x)$. This operator is Hermitian

1. provided the wavefunction $\psi(x)$ is normalized
2. provided the wavefunction $\psi(x)$ and derivate $\psi'(x)$ are continuous everywhere
3. provided the wavefunction $\psi(x)$ vanishes as $x \to \pm\infty$
4. by its very definition

Q.No:15 TIFR-2021

A unitary matrix U is expressed in terms of a Hermitian matrix H, such that

$$U = e^{i\pi H/2}$$

If the matrix H is given by

$$H = \sqrt{3}\begin{pmatrix} 1/3 & 0 & \sqrt{2}/3 \\ 0 & 1/\sqrt{3} & 0 \\ \sqrt{2}/3 & 0 & -1/3 \end{pmatrix}$$

then U will have the form

1. $\begin{pmatrix} i/\sqrt{3} & 0 & i\sqrt{2}/\sqrt{3} \\ 0 & i & 0 \\ i\sqrt{2}/\sqrt{3} & 0 & -i/\sqrt{3} \end{pmatrix}$
2. $\begin{pmatrix} \sqrt{3} & 0 & \sqrt{6} \\ 0 & 3\sqrt{3} & 0 \\ \sqrt{6} & 0 & -\sqrt{3} \end{pmatrix}$
3. $\begin{pmatrix} i\sqrt{3} & 1/\sqrt{3} & \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} & i & 1/\sqrt{3} \\ \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & i/\sqrt{3} \end{pmatrix}$
4. $\begin{pmatrix} 3\sqrt{3}i & \sqrt{3} & 3/2 \\ \sqrt{3} & i & 0 \\ \sqrt{2}/\sqrt{3} & 0 & 3\sqrt{3}i \end{pmatrix}$

Q.No:16 TIFR-2022

Consider a set of three 3-dimensional vectors

$$A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

These vectors undergo a linear transformation

$$A \to A' = \mathbb{M}A \quad B \to B' = \mathbb{M}B \quad C \to C' = \mathbb{M}C$$

where $\mathbb{M}$ is given by

$$\mathbb{M} = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

The volume of a parallelopiped whose sides are given by the transformed vectors A', B' and C' is

1. 8
2. 4
3. 2
4. 16

Q.No:17 TIFR-2022

Consider the inner product in the space of normalisable functions defined on the interval [-1,1]

$$\langle f|g\rangle = \int_{-1}^{1} dx\ (1+x^2)\ f(x)\ g(x)$$

The projection of the vector 1 along the vector x^2 is

1. $\frac{14}{9}x^2$
2. $\frac{16}{15}\sqrt{\frac{35}{24}}x^2$
3. $\frac{16}{15}x^2$
4. $\sqrt{\frac{35}{24}}x^2$

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Q.No:18 TIFR-2022

A particle is confined to a one-dimensional lattice with a lattice spacing δ. In the position space, the Hamiltonian operator for this particle is given by the matrix

$$H = E_0 \begin{pmatrix} ... & ... & 0 & 0 & 0 & 0 \\ ... & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & ... \\ 0 & 0 & 0 & 0 & ... & ... \end{pmatrix}$$

Noting that it commutes with the generator T of translations

$$T = \begin{pmatrix} ... & ... & 0 & 0 & 0 & 0 \\ ... & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & ... & ... \\ 0 & 0 & 0 & 0 & ... & ... \end{pmatrix}$$

where $T = e^{iP\,\delta/\hbar}$ in terms of the momentum operator P, the energy of a state with momentum p will be

1. $4E_0\ \sin^2(p\delta/2\hbar)$
2. $E_0\ \cos(p\delta/\hbar)$
3. $E_0\ \sin(p\delta/\hbar)$
4. $E_0\ (p\delta/2\hbar)^2$

Q.No:19 TIFR-2023

Consider a symmetric matrix

$$M = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \end{pmatrix}$$

An orthogonal matrix O which can be diagonalize this matrix by an orthogonal transformation $O^T MO$ is given by $O =$

1. $\begin{pmatrix} \sqrt{3/2} & 0 & \sqrt{1/3} \\ 0 & 1 & 0 \\ \sqrt{1/3} & 0 & -\sqrt{2/3} \end{pmatrix}$
2. $\begin{pmatrix} 1/\sqrt{2} & 0 & i/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \end{pmatrix}$
3. $\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$
4. $\begin{pmatrix} \sqrt{1/3} & 0 & \sqrt{3/2} \\ 0 & 1 & 0 \\ \sqrt{2/3} & 0 & -\sqrt{1/3} \end{pmatrix}$

Q.No:20 TIFR-2024

Consider $\hat{x}$ and $\hat{p}_x$ as the quantum mechanical position and linear momentum operators with eigenstates $|x\rangle$ and $|p_x\rangle$, and eigenvalues x and p_x, respectively.

The eigenvalue of $\hat{x}$ acting on the state

$$|\psi\rangle = e^{\frac{i\hat{p}_x a}{2\hbar}}|x\rangle$$

is

1. $x + \frac{a}{2}$
2. $x - \frac{a}{2}$
3. $x + a$
4. $x - a$

Q.No:21 TIFR-2024

Consider the following matrix

$$M = \begin{pmatrix} 1 & 5 & -7 & 1 \\ 1 & 0 & 2 & 2 \\ 9 & -1 & 3 & 1 \\ 9 & 6 & -7 & -4 \end{pmatrix}$$

What is $\det(e^M)$?

(a) e

(b) e^{1210}

(c) 1

(d) e^{-1210}

Q.No:22 TIFR-2025

The $n \times n$ $(n > 4)$ matrix M, with all entries equal to 1, has:

1. Precisely $n-1$ degenerate eigenvalues and one other non-degenerate eigenvalue
2. Precisely $n-2$ degenerate eigenvalues and two other non-degenerate eigenvalues
3. Precisely 2 degenerate eigenvalues and $n-2$ other non-degenerate eigenvalues
4. No degenerate eigenvalues

Q.No:23 TIFR-2026

Consider a quantum particle moving in one spatial dimension with the Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$ Let $|\psi(t)\rangle$ be the state of the system where $|\psi(0)\rangle$ is a normalized eigenstate of this Hamiltonian. What is the value of $\langle\psi(t)|\hat{x}\hat{p}|\psi(t)\rangle$? (Hint: Consider $\frac{d\langle\psi(t)|\hat{x}^2|\psi(t)\rangle}{dt}$.)

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(a) $\frac{i\hbar}{2}$

(b) $-i\hbar$

(c) $\hbar$

(d) 0

Q.No:24 TIFR-2026

The Hamiltonian of a quantum system in the basis states, $|1\rangle$, $|2\rangle$, and $|3\rangle$ is given by the matrix: $\omega_0 \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$ If the system is in the state $|\psi(0)\rangle = \frac{|1\rangle - |3\rangle}{\sqrt{2}}$ at $t = 0$, the probability of the system to be in state $|2\rangle$ at time t is given by:

(a) 0

(b) $\sin^2(2\omega_0 t)$

(c) $\sin^2(\omega_0 t)$

(d) $\sin^2\left(\frac{\omega_0 t}{2}\right)$

Q.No:25 TIFR-2026

The inner product of two complex functions $f(x)$, $g(x)$, over the interval $x \in [0, 1]$ is defined as:

$$\langle f|g\rangle = \int_0^1 dx\, x(1-x) f^*(x) g(x)$$

The adjoint, $\hat{O}^\dagger$, of a linear operator $\hat{O}$ is defined by the equation: $\langle f|\hat{O}g\rangle = \langle \hat{O}^\dagger f|g\rangle$ What is the adjoint of the operator $\hat{O} = \frac{d}{dx}$? Assume that both $f(x)$ and $g(x)$ are differentiable.

(a) $\frac{2x-1}{x(1-x)} - \frac{d}{dx}$

(b) $-\frac{d}{dx}$

(c) $\frac{1}{x(1-x)} \frac{d}{dx}$

(d) $-\frac{1}{x(1-x)} \frac{d}{dx}$

Question No.	Entrance with year	Answer
1	TIFR-2012	c
2	TIFR-2013	d
3	TIFR-2013	d
4	TIFR-2014	a
5	TIFR-2015	b
6	TIFR-2016	c
7	TIFR-2016	a
8	TIFR-2017	c
9	TIFR-2017	50
10	TIFR-2017	b
11	TIFR-2018	a
12	TIFR-2019	a
13	TIFR-2020	a
14	TIFR-2020	c
15	TIFR-2021	a
16	TIFR-2022	a
17	TIFR-2022	a
18	TIFR-2022	a
19	TIFR-2023	c
20	TIFR-2024	2
21	TIFR-2024	c
22	TIFR-2025	a
23	TIFR-2026	a
24	TIFR-2026	a
25	TIFR-2026	a

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