Other Quantum Mechanical Questions CSIR, JEST & TIFR

https://snboseplc.com/other-quantum-mechanics-questions-csir-jest-tifr/

1 CSIR June-2015

In a two-state system, the transition rate of a particle from state 1 to state 2 is t_{12}, and the transition rate from state 2 to state 1 is t_{21}. In the steady state, the probability of finding the particle in state 1 is

A. $\frac{t_{21}}{t_{12}+t_{21}}$

B. $\frac{t_{12}}{t_{12}+t_{21}}$

C. $\frac{t_{12}-t_{21}}{t_{12}+t_{21}}$

D. $\frac{t_{12}t_{21}}{t_{21}+t_{21}}$

2 CSIR June-2015

The Dirac Hamiltonian $H = c\vec{\alpha}\cdot\vec{p}+\beta mc^2$ for a free electron corresponds to the classical relation $E^2 = p^2c^2 + m^2c^4$. The classical energy-momentum relation of a particle of charge q in an electromagnetic potential $(\phi, \vec{A})$ is $(E-q\phi)^2 = c^2\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2 + m^2c^4$.

Therefore, the Dirac Hamiltonian for an electron in an electromagnetic field is

A. $c\vec{\alpha}\cdot\vec{p}+\frac{e}{c}\vec{A}\cdot\vec{A}+\beta mc^2-e\phi$

B. $c\vec{\alpha}\cdot\left(\vec{p}+\frac{e}{c}\vec{A}\right)+\beta mc^2+e\phi$

C. $c\left(\vec{\alpha}\cdot\vec{p}+e\phi+\frac{e}{c}|\vec{A}|\right)+\beta mc^2$

D. $c\vec{\alpha}\cdot\left(\vec{p}+\frac{e}{c}\vec{A}\right)+\beta mc^2-e\phi$

3 CSIR June-2016

Using dimensional analysis, Planck defined a characteristic temperature T_P from powers of the gravitational constant G, Planck's constant h, Boltzmann constant k_B and the speed of light c in vacuum. The expression for T_P is proportional to

A. $\sqrt{\frac{hc^5}{k_B^2G}}$

B. $\sqrt{\frac{hc^3}{k_B^2G}}$

C. $\sqrt{\frac{G}{hc^4k_B^2}}$

D. $\sqrt{\frac{hk_B^2}{Gc^3}}$

4 CSIR Dec-2016

Consider the operator $\vec{\pi} = \vec{p}-q\vec{A}$, where $\vec{p}$ is the momentum operator, $\vec{A} = (A_x, A_y, A_z)$ is the vector potential and q denotes the electric charge. If $\vec{B} = (B_x, B_y, B_z)$ denotes the magnetic field, the z-component of the vector operator $\vec{\pi}\times\vec{\pi}$ is

A. $iq\hbar B_z + q(A_xp_y - A_yp_x)$

B. $-iq\hbar B_z - q(A_xp_y - A_yp_x)$

C. $-iq\hbar B_z$

D. $iq\hbar B_z$

5 CSIR Dec-2016

The dynamics of a free relativistic particle of mass m is governed by the Dirac Hamiltonian $H = c\vec{\alpha}\cdot\vec{p}+\beta mc^2$, where $\vec{p}$ is the momentum operator and $\vec{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ and β are four 4×4 Dirac matrices. The acceleration operator can be expressed as

A. $\frac{2ic}{\hbar}(c\vec{p}-\vec{\alpha}H)$

B. $2ic^2\vec{\alpha}\beta$

C. $\frac{ic}{\hbar}H\vec{\alpha}$

D. $-\frac{2ic}{\hbar}(c\vec{p}+\vec{\alpha}H)$

6 CSIR Dec-2016

A particle of charge q in one dimension is in a simple harmonic potential with angular frequency ω. It is subjected to a time-dependent electric field $E(t) = Ae^{-(t/\tau)^2}$, where A and τ are positive constants and $\omega\tau \gg 1$. If in the distant past $t\to-\infty$ the particle was in its ground state, the probability that it will be in the first excited state as $t\to+\infty$ is proportional to

A. $e^{-\frac{1}{2}(\omega\tau)^2}$

B. $e^{\frac{1}{2}(\omega\tau)^2}$

C. 0

D. $\frac{1}{(\omega\tau)^2}$

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7 CSIR June-2017

A constant perturbation H' is applied to a system for time Δt (where $H'\Delta t \ll \hbar$) leading to a transition from a state with energy E_i to another with energy E_f. If the time of application is doubled, the probability of transition will be

A. unchanged

B. doubled

C. quadrupled

D. halved

8 CSIR June-2017

In the usual notation $|nlm\rangle$ for the states of a hydrogen like atom, consider the spontaneous transitions $|210\rangle \rightarrow |100\rangle$ and $|310\rangle \rightarrow |100\rangle$. If t_1 and t_2 are the lifetimes of the first and the second decaying states respectively, then the ratio $\frac{t_1}{t_2}$ is proportional to

A. $\left(\frac{32}{27}\right)^3$

B. $\left(\frac{27}{32}\right)^3$

C. $\left(\frac{2}{3}\right)^3$

D. $\left(\frac{3}{2}\right)^3$

9 CSIR Dec-2017

Consider a system of identical atoms in equilibrium with blackbody radiation in a cavity at temperature T. The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1\rangle$ are P_0 and P_1, respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\rightarrow1}$ and $W_{1\rightarrow0}$ denote, respectively, the squares of the matrix elements corresponding to the atomic transitions $|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$. Which of the following equations hold in equilibrium?

A. $P_0 n W_{0\rightarrow1} = P_1 W_{1\rightarrow0}$

B. $P_0 W_{0\rightarrow1} = P_1 n W_{1\rightarrow0}$

C. $P_0 n W_{0\rightarrow1} = P_1 W_{1\rightarrow0} - P_1 n W_{1\rightarrow0}$

D. $P_0 n W_{0\rightarrow1} = P_1 W_{1\rightarrow0} + P_1 n W_{1\rightarrow0}$

10 CSIR June-2019

A charged, spin-less particle of mass m is subjected to an attractive potential $V(x,y,z) = \frac{1}{2}k(x^2+y^2+z^2)$, where k is a positive constant. Now a perturbation in the form of a weak magnetic field $\mathbf{B} = B_0\hat{k}$ (where B_0 is a constant) is switched on. Into how many distinct levels will the second excited state of the unperturbed Hamiltonian split?

A. 5

B. 4

C. 2

D. 1

11 CSIR June-2019

The range of the inter-atomic potential in gaseous hydrogen is approximately 5 Angstrom. In thermal equilibrium, the maximum temperature for which the atom-atom scattering is dominantly s-wave, is

A. $500K$

B. $1K$

C. $100K$

D. $1mK$

12 CSIR Feb-2022

The energies of a two-state quantum system are E_0 and $E_0+\alpha\hbar$, (where $\alpha > 0$ is a constant) and the corresponding normalized state vectors are $|0\rangle$ and $|1\rangle$, respectively. At time $t = 0$, when the system is in the state $|0\rangle$, the potential is altered by a time independent term V such that $\langle 1|V|0\rangle = \hbar\alpha/10$. The transition probability to the state $|1\rangle$ at times $t \ll 1/\alpha$, is

A. $\alpha^2t^2/25$

B. $\alpha^2t^2/50$

C. $\alpha^2t^2/100$

D. $\alpha^2t^2/200$

13 CSIR Feb-2022

The $|3,0,0\rangle$ state in the standard notation $|n,l,m\rangle$ of the H-atom in the non-relativistic theory decays to the state $|1,0,0\rangle$ via two dipole transition. The transition route and the corresponding probability are

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A. $|3,0,0\rangle \to |2,1,-1\rangle \to |1,0,0\rangle$ and 1/4

B. $|3,0,0\rangle \to |2,1,1\rangle \to |1,0,0\rangle$ and 1/4

C. $|3,0,0\rangle \to |2,1,0\rangle \to |1,0,0\rangle$ and 1/3

D. $|3,0,0\rangle \to |2,1,0\rangle \to |1,0,0\rangle$ and 2/3

14 CSIR Sep-2022

At time $t = 0$, a particle in the ground state of the Hamiltonian

$$H(t) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x \sin\left(\frac{\omega t}{2}\right)$$

where λ, ω and m are positive constants. To $O(\lambda^2)$, the probability that at $t = 2\pi/\omega$, the particle would be in the first excited state of $H(t = 0)$ is

A. $\frac{9\lambda^2}{16m\hbar\omega^3}$

B. $\frac{9\lambda^2}{8m\hbar\omega^3}$

C. $\frac{16\lambda^2}{9m\hbar\omega^3}$

D. $\frac{8\lambda^2}{9m\hbar\omega^3}$

15 CSIR June-2026

A particle is in the ground state of a one-dimensional infinite well potential with walls at $x = 0$ and $x = a$. The wall at $x = a$ is moved to $x = 2a$ in two separate, sudden and adiabatic protocols so that $\langle E\rangle_s$ and $\langle E\rangle_a$ are the respective expectation values of the energy in the final states. Then the ratio

$$\frac{\langle E\rangle_s}{\langle E\rangle_a}$$

is:

1. 4 : 1
2. 2 : 1
3. 1 : 1
4. 1 : 4

1 JEST-2017

If $\hat{x}(t)$ be the position operator at a time t in the Heisenberg picture for a particle described by the Hamiltonian, $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, what is $e^{i\omega t}\langle 0|\hat{x}(t)\hat{x}(0)|0\rangle$ in units of $\frac{\hbar}{2m\omega}$ where $|0\rangle$ is the ground state?

2 JEST-2017

The normalized eigenfunctions and eigenvalues of the Hamiltonian of a particle confined to move between $0 \le x \le a$ in one dimension are

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$$

and

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

respectively. Here $n = 1, 2, 3, \cdots$. Suppose the state of the particle is

$$\psi(x) = A\sin\left(\frac{\pi x}{a}\right)\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]$$

where A is the normalization constant. If the energy of the particle is measured, the probability to get the result as $\frac{\pi^2\hbar^2}{2ma^2}$ is $x/100$. What is the value of x?

3 JEST-2021

Consider a 4-dimensional vector space V that is a direct product of two 2-dimensional vector spaces V_1 and V_2. A linear transformation H acting on V is specified by the direct product of linear transformations H_1 and H_2 acting on V_1 and V_2, respectively. In a particular basis,

$$H_1 = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}, H_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix},$$

what is the lowest eigenvalue of H?

A. 1

B. $\frac{3}{2}$

C. $3 - \sqrt{5}$

D. $\frac{1}{2}(3 - \sqrt{5})$

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1 TIFR-2012

In a scanning tunnelling microscope, a fine Platinum needle is held close to a metallic surface in vacuum and electrons are allowed to tunnel across the tiny gap δ between the surface and the needle. The tunnelling current I is related to the gap δ, through positive constants a and b, as

A. $I = a - b\delta$

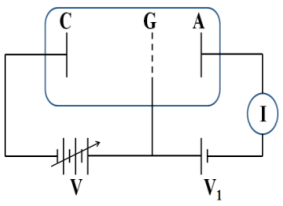
B. $I = a + b\delta$

C. $\log I = a - b\delta$

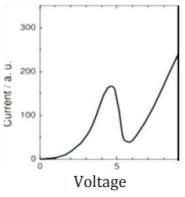
D. $\log I = a + b\delta$

2 TIFR-2016

In the experiment shown in Figure (i) below, the emitted electrons from the cathode (C) are made to pass through the mercury vapor filled in the tube by accelerating them using a grid (G) at potential V, positive w.r.t. the cathode. The electrons are collected by the anode (A).



(i)



(ii)

The variation of electron current (I) as a function of V is given in figure (ii). The shape of this curve must be interpreted as due to

A. ionization of mercury atoms.

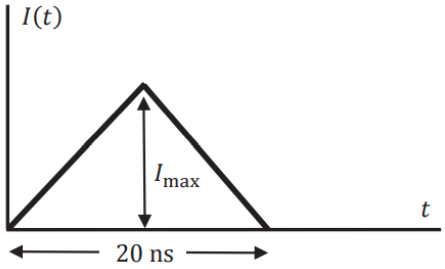
B. an emission line from mercury atoms.

C. attachment of electrons to mercury atoms.

D. resonant backscattering of electrons to cathode from grid.

3 TIFR-2017

A photomultiplier tube is used to detect identical light pulses each of which consists of a fixed number of photons. The photoelectric efficiency is 10%, i.e. a photon has 10% probability of causing the emission of a detectable photoelectron. The photomultiplier gain is 10^6.



The typical output current, as a function of time, is shown by the figure below for a few pulses, where $I_{\max}$ is 80 μA. It follows that the number of photons in each pulse is

A. 5×10^6

B. 5

C. 800

D. 50

4 TIFR-2018

A particle in a one-dimensional harmonic oscillator potential is described by a wave-function $\psi(x,t)$. If the wavefunction changes to $\psi(\lambda x, t)$ then the expectation value of kinetic energy T and the potential energy V will change, respectively, to

A. $\lambda^2 T$ and V/λ^2

B. T/λ^2 and V/λ^2

C. T/λ^2 and $\lambda^2 V$

D. $\lambda^2 T$ and $\lambda^2 V$

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Question No.	**Entrance with year**	
1	CSIR June-2015	Option A
2	CSIR June-2015	Option D
3	CSIR June-2016	Option A
4	CSIR Dec-2016	Option D
5	CSIR Dec-2016	Option A
6	CSIR Dec-2016	Option A
7	CSIR June-2017	Option C
8	CSIR June-2017	Option A
9	CSIR Dec-2017	Option D
10	CSIR June-2019	Option A
11	CSIR June-2019	Option C
12	CSIR Feb-2022	Option C
13	CSIR Feb-2022	Option C
14	CSIR Sep-2022	Option D
15	CSIR June-2026	Option 1
1	JEST-2017	1
2	JEST-2017	50
3	JEST-2021	Option C
1	TIFR-2012	Option c
2	TIFR-2016	Option b
3	TIFR-2017	Option d
4	TIFR-2018	Option a

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