Perturbation Theory and Fine Structure JEST & TIFR

https://snboseplc.com/perturbation-theory-jest-tifr/

JEST Questions

Q.No:1 JEST-2012

Consider a system of two spin-1/2 particles with total spin $\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2$, where $\mathbf{s}_1$ and $\mathbf{s}_2$ are in terms of Pauli matrices σ_i. The spin triplet projection operator is

(a) $\frac{1}{4} + \mathbf{s}_1 \cdot \mathbf{s}_2$
(b) $\frac{3}{4} - \mathbf{s}_1 \cdot \mathbf{s}_2$
(c) $\frac{3}{4} + \mathbf{s}_1 \cdot \mathbf{s}_2$
(d) $\frac{1}{4} - \mathbf{s}_1 \cdot \mathbf{s}_2$

Q.No:2 JEST-2013

A simple model of a helium-like atom with electron-electron interaction is replaced by Hooke's law force is described by hamiltonian $\frac{-\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) + \frac{1}{2}m\omega^2(r_1^2 + r_2^2) - \frac{\lambda}{4}m\omega^2|\vec{r}_1 - \vec{r}_2|^2$. What is the exact ground state energy?

(a) $E = 3/2\hbar\omega(1 + \sqrt{1+\lambda})$
(b) $E = 3/2\hbar\omega(1 + \sqrt{\lambda})$
(c) $E = 3/2\hbar\omega\sqrt{1-\lambda}$
(d) $E = 3/2\hbar\omega(1 + \sqrt{1-\lambda})$

Q.No:3 JEST-2015

Consider a spin-$\frac{1}{2}$ particle characterized by the Hamiltonian $H = \omega S_z$. Under a perturbation $H' = gS_x$, the second order correction to the ground state energy is given by,

(a) $-\frac{g^2}{4\omega}$
(b) $\frac{g^2}{4\omega}$
(c) $-\frac{g^2}{2\omega}$
(d) $\frac{g^2}{2\omega}$

Q.No:4 JEST-2015

A particle of mass m is confined in a potential well given by $V(x) = 0$ for $\frac{-L}{2} < x < \frac{L}{2}$ $L/2$ and $V(x) = \infty$ elsewhere. A perturbing potential $H'(x) = ax$ has been applied to the system. Let the first and second order corrections to the ground state be $E_0^{(1)}$ and $E_0^{(2)}$, respectively. Which one of the following statements is correct?

(a) $E_0^{(1)}0$
(b) $E_0^{(1)} = 0$ and $E_0^{(2)} > 0$
(c) $E_0^{(1)} > 0$ and $E_0^{(2)} < 0$
(d) $E_0^{(1)} = 0$ and $E_0^{(2)} < 0$

Q.No:5 JEST-2016

Consider a quantum particle of mass m in one dimension in an infinite potential well, *i.e.*, $V(x) = 0$ for $-a/2 < x < a/2$, and $V(x) = \infty$ for $|x| \geq a/2$. A small perturbation, $V'(x) = 2\epsilon|x|/a$, is added. The change in the ground state energy to $O(\epsilon)$ is:

(a) $\frac{\epsilon}{2\pi^2}(\pi^2 - 4)$
(b) $\frac{\epsilon}{2\pi^2}(\pi^2 + 4)$
(c) $\frac{\epsilon\pi^2}{2}(\pi^2 + 4)$
(d) $\frac{\epsilon\pi^2}{2}(\pi^2 - 4)$

Q.No:6 JEST-2017

Suppose the spin degrees of freedom of a 2-particle system can be described by a 21-dimensional Hilbert subspace. Which among the following could be the spin of one of the particles?

(a) $\frac{1}{2}$
(b) 3
(c) $\frac{3}{2}$
(d) 2

Q.No:7 JEST-2017

A particle is described by the following Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 + \lambda\hat{x}^4,$$

where the quartic term can be treated perturbatively. If ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state respectively, what is the fraction $\Delta E_1/\Delta E_0$?

Q.No:8 JEST-2018

What is the difference between the maximum and the minimum eigenvalues of a system of two electrons whose Hamiltonian is $H + J\vec{S}_1 \cdot \vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the corresponding spin angular momentum operators of the two electrons?

(a) $J/4$
(b) $J/2$
(c) $3J/4$
(d) J

Q.No:9 JEST-2018

Suppose the spin degree of freedom of two particles (nonzero rest mass and nonzero spin) is described completely by a Hilbert space of dimension twenty one. Which of the following could be the spin of one of the particles?

(a) 2
(b) 3/2
(c) 1
(d) 1/2

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Q.No:10 JEST-2018

A harmonic oscillator has the following Hamiltonian

$$H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

It is perturbed with a potential $V = \lambda\hat{x}^4$. Some of the matrix elements of $\hat{x}^2$ in terms of its expectation value in the ground state are given as follows:

$$\begin{aligned}\langle 0|\hat{x}^2|0\rangle &= C\\ \langle 0|\hat{x}^2|2\rangle &= \sqrt{2}C\\ \langle 1|\hat{x}^2|1\rangle &= 3C\\ \langle 1|\hat{x}^2|3\rangle &= \sqrt{6}C\end{aligned}$$

where $|n\rangle$ is the normalized eigenstate of H_0 corresponding to the eigenvalue $E_n = \hbar\omega(n + 1/2)$. Suppose ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state, respectively. What is the fraction $\Delta E_1/\Delta E_0$?

Q.No:11 JEST-2018

Consider two coupled harmonic oscillators of mass m each. The Hamiltonian describing the oscillators is

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}m\omega^2(\hat{x}_1^2 + \hat{x}_2^2 + (\hat{x}_1 - \hat{x}_2)^2).$$

The eigenvalues of $\hat{H}$ are given by (with n_1 and n_2 being non-negative integers)

(a) $E_{n_1,n_2} = \hbar\omega(n_1 + n_2 + 1)$
(b) $E_{n_1,n_2} = \hbar\omega(n_1 + \frac{1}{2}) + \frac{1}{\sqrt{3}}\hbar\omega(n_2 + \frac{1}{2})$
(c) $E_{n_1,n_2} = \hbar\omega(n_1 + \frac{1}{2}) + \sqrt{3}\hbar\omega(n_2 + \frac{1}{2})$
(d) $E_{n_1,n_2} = \frac{1}{\sqrt{3}}\hbar\omega(n_1 + n_2 + 1)$

Q.No:12 JEST-2019

Consider a quantum particle of mass m and charge e moving in a two dimensional potential given as:

$$V(x, y) = \frac{k}{2}(x - y)^2 + k(x + y)^2.$$

The particle is also subject to an external electric field $\vec{E} = \lambda(\hat{i} - \hat{j})$, where λ is a constant. $\hat{i}$ and $\hat{j}$ corresponds to unit vectors along x and y directions, respectively. Let E_1 and E_0 be the energies of the first excited state and ground state, respectively. What is the value of $E_1 - E_0$?

(a) $\hbar\sqrt{2k/m}$
(b) $\hbar\sqrt{2k/m} + e\lambda^2$
(c) $3\hbar\sqrt{2k/m}$
(d) $3\hbar\sqrt{2k/m} + e\lambda^2$

Q.No:13 JEST-2023

Consider a two-level quantum system described by the Hamiltonian:

$$H = H_0 + H'$$

where

$$H_0 = \alpha\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad H' = \epsilon\Gamma\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

H' is a small perturbation to the free Hamiltonian H_0. ϵ is a small positive dimensionless number, while α, ω and Γ have dimensions of energy and are positive quantities. If we treat this problem perturbatively in the parameter ϵ, which of the following statements about the corrections to ground state energy is true?

1) First-order correction is $\epsilon\Gamma$; second-order correction is $-\frac{\epsilon^2\Gamma^2}{2\omega}$
2) First-order correction is $\epsilon\Gamma$; second-order correction is 0.
3) First-order correction is 0; second-order correction is $-\frac{\epsilon^2\Gamma^2}{2\omega}$
4) First-order correction is 0; second-order correction is $\frac{\epsilon^2\Gamma^2}{2\omega}$

Q.No:14 JEST-2025

For a one-dimensional simple harmonic oscillator with mass m and angular frequency ω, consider a perturbation λx^4 in the Hamiltonian ($\lambda > 0$). What is the lowest order correction to the ground state energy?

(The position operator expressed in terms of the raising and lowering operators is

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^\dagger).$$

)

1) $\frac{3\lambda}{2}\left(\frac{\hbar}{m\omega}\right)^2$
2) $\frac{5\lambda}{4}\left(\frac{\hbar}{m\omega}\right)^2$
3) $\frac{3\lambda}{4}\left(\frac{\hbar}{m\omega}\right)^2$
4) $\frac{5\lambda}{2}\left(\frac{\hbar}{m\omega}\right)^2$

Q.No:15 JEST-2026

A system of two spin-$\frac{1}{2}$ particles is in the state $|\uparrow\downarrow\rangle$ for time $t < 0$. At $t = 0$, an interaction Hamiltonian

$$H = CS^{\hat{(1)}} \cdot S^{\hat{(2)}}$$

is turned on, where C is a constant of appropriate dimension and $S^{\hat{(1)}}$ and $S^{\hat{(2)}}$ are the spin angular momentum operators of the first and second particle respectively. What is the probability in percentage to find the system in the state $|\downarrow\uparrow\rangle$ at time

$$t = \frac{2\pi}{3C\hbar}\ ?$$

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Q.No:1 TIFR-2014

A particle of mass m and charge e is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

(a) make a transition to any harmonic oscillator state with $x = -eE/k$ as origin without emitting any photon.

(b) make a transition to any harmonic oscillator state with $x = 0$ as origin and absorb a photon.

(c) settle into the harmonic oscillator ground state with $x = 0$ as origin after absorbing a photon.

(d) oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

Q.No: 2 TIFR-2016

Consider two spin-1/2 identical particles A and B, separated by a distance r, interacting through a potential

$$V(r) = \frac{V_0}{r}\vec{S}_A.\vec{S}_B$$

where V_0 is a positive constant and the spins are $\vec{S}_{A,B} = \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of the Pauli spin matrices. The expectation values of this potential in the spin-singlet and triplet states are

(a) Singlet: $\frac{V_0}{3r}$, Triplet: $\frac{V_0}{r}$

(b) Singlet: $-\frac{3V_0}{r}$, Triplet: $\frac{V_0}{r}$

(c) Singlet: $\frac{3V_0}{r}$, Triplet: $-\frac{V_0}{r}$

(d) Singlet: $-\frac{V_0}{r}$, Triplet: $\frac{3V_0}{r}$

Q.No: 3 TIFR-2017

A quantum mechanical system which has stationary states $|1\rangle, |2\rangle$ and $|3\rangle$, corresponding to energy levels 0 eV, 1 eV and 2 eV respectively, is perturbed by a potential of the form

$$\hat{V} = \varepsilon|1\rangle\langle 3| + \varepsilon|3\rangle\langle 1|$$

where, in eV, $0 < \varepsilon \ll 1$.

The new ground state, correct to order ε, is approximately.

(a) $\left(1 - \frac{\varepsilon}{2}\right)|1\rangle + \frac{\varepsilon}{2}|3\rangle$

(b) $|1\rangle + \frac{\varepsilon}{2}|2\rangle - \varepsilon|3\rangle$

(c) $|1\rangle + \frac{\varepsilon}{2}|3\rangle$

(d) $|1\rangle - \frac{\varepsilon}{2}|3\rangle$

Q.No: 4 TIFR-2018

A particle of mass m moves in a two-dimensional space (x, y) under the influence of a Hamiltonian

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{4}m\omega^2(5x^2 + 5y^2 + 6xy)$$

Find the ground state energy of this particle in a quantum-mechanical treatment.

Q.No: 5 TIFR-2019

A system of two spin-1/2 particles 1 and 2 has the Hamiltonian

$$\hat{H} = \epsilon_0 \hat{h}_1 \otimes \hat{h}_2$$

where

$$\hat{L}_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \hat{L}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and ϵ_0 is a constant with the dimension of energy. The ground state of this system has energy

(a) $\sqrt{2}\epsilon_0$

(b) 0

(c) $-2\epsilon_0$

(d) $-4\epsilon_0$

Q.No: 6 TIFR-2021

What are the energy eigenvalues for relative motion in one-dimension of a two-body simple quantum harmonic oscillator (each body having mass m) with the following Hamiltonian?

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{m} + \frac{1}{2}m\omega^2(x_1 - x_2)^2$$

(a) $\sqrt{2}\left(n + \frac{1}{2}\right)\hbar\omega$

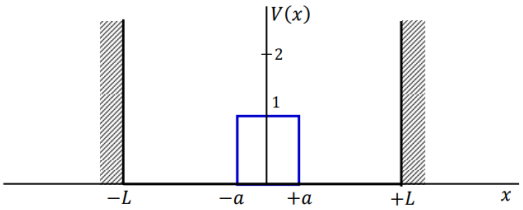
(b) $\left(n + \frac{1}{2}\right)\hbar\omega$

(c) $\frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right)\hbar\omega$

(d) $\sqrt{\frac{3}{2}}\left(n + \frac{1}{2}\right)\hbar\omega$

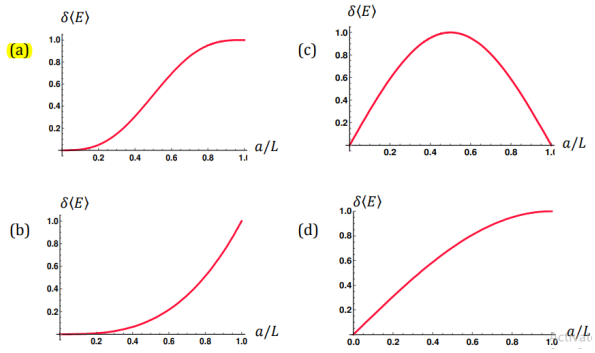
Q.No: 7 TIFR-2021

A particle of mass m, confined in a one-dimensional box between $x = -L$ and $x = L$, is in its first excited quantum state. Now, a rectangular potential barrier of height $V(x) = 1$ and extending from $x = -a$ to $x = a$ is suddenly switched on, as shown in the figure below.



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Which of the following curves most closely represents the resulting change in average energy $\delta\langle E\rangle$ of the system when plotted as a function of a/L, immediately after the barrier is created?



Q.No: 8 TIFR-2025

Consider a particle with mass m in a quantum harmonic oscillator potential with a frequency ω, such that its Hamiltonian is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}$$

The Hamiltonian is perturbed by adding a term to the potential

$$\Delta\hat{H} = \lambda \sin\hat{x}$$

where λ is small compared to $\hbar\omega$. The relative change in the ground state energy, to the leading order in $\lambda/(\hbar\omega)$, is given by:

a) $O\left(\frac{\lambda^2}{(\hbar\omega)^2}\right)$
b) $O\left(\frac{\lambda}{\hbar\omega}\right)$
c) $O(1)$
d) The ground state energy does not change

Question No.	Entrance with year	answer
1	JEST-2012	Option (c)
2	JEST-2013	Option (d)
3	JEST-2015	Option (a)
4	JEST-2015	Option (d)
5	JEST-2016	Option (a)
6	JEST-2017	Option (b)
7	JEST-2017	5
8	JEST-2018	Option (d)
9	JEST-2018	Option (c)
10	JEST-2018	5
11	JEST-2018	Option (c)
12	JEST-2019	Option (a)
13	JEST-2023	Option 3
14	JEST-2025	Option 3
15	JEST-2026	75
1	TIFR-2014	Option (b)
2	TIFR-2016	Option (b)
3	TIFR-2017	Option (d)
4	TIFR-2018	-
5	TIFR-2019	Option (c)
6	TIFR-2021	Option (a)
7	TIFR-2021	Option (a)
8	TIFR-2025	Option (a)

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