Perturbation Theory and Fine Structure GATE & CSIR

https://snboseplc.com/perturbation-theory-and-fine-structure-gate-csir/

GATE Questions

Q.No:1 GATE-2012

Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$ where $\delta \ll 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are

(A) 1 and $(1 + 2\delta)$
(B) $(1 + \delta)$ and $(1 - \delta)$
(C) $(1 + 2\delta)$ and $(1 - 2\delta)$
(D) $(1 + \delta)$ and $(1 - 2\delta)$

Q.No:2 GATE-2013

To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

we add a small perturbation given by

$$\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix},$$

where ε is a small quantity.
The ground state eigenvector of the unperturbed Hamiltonian is

(A) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$
(B) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$
(C) $(0, 0, 1)$
(D) $(1, 0, 0)$

Q.No:3 GATE-2013

To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

we add a small perturbation given by

$$\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix},$$

where ε is a small quantity. A pair of eigenvalues of the perturbed Hamiltonian, using first order perturbation theory, is

(A) $3 + 2\varepsilon, 7 + 2\varepsilon$
(B) $3 + 2\varepsilon, 2 + \varepsilon$
(C) $3, 7 + 2\varepsilon$
(D) $3, 2 + 2\varepsilon$

Q.No:4 GATE-2014

If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does NOT commute with

(A) S_z
(B) L^2
(C) S^2
(D) $(\vec{L} + \vec{S})^2$

Q.No:5 GATE-2014

A particle is confined to a one dimensional potential box with the potential

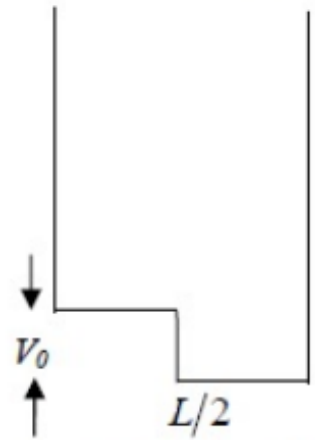
$$\begin{aligned} V(x) &= 0, \quad 0 < x < a \\ &= \infty, \quad \text{otherwise} \end{aligned}$$

If the particle is subjected to a perturbation, within the box, $W = \beta x$, where β is a small constant, the first order correction to the ground state energy is

(A) 0
(B) $\alpha\beta/4$
(C) $\alpha\beta/2$
(D) $\alpha\beta$

Q.No:6 GATE-2015

A particle is confined in a box of length L as shown below.



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

(A) $E = \frac{\hbar^2\pi^2}{2mL^2} + V_0$
(B) $E = \frac{\hbar^2\pi^2}{2mL^2} - \frac{V_0}{2}$

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(C) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{4}$
(D) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{2}$

Q.No:7 GATE-2015

Let the Hamiltonian for two spin-1/2 particles of equal masses m, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be $H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2(r_1^2 + r_2^2) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1eV$ and $k = 0.2eV$. If the ground state has net spin zero, then the energy (in eV) is _____.

Q.No:8 GATE-2016

A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E} = E_0\hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent n is _____.

Q.No:9 GATE-2016

If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle \vec{s}_1, \vec{s}_2 \rangle$ for the ground state is
(A) $-\frac{3}{2}\hbar^2$
(B) $-\frac{3}{4}\hbar^2$
(C) 0
(D) $\frac{1}{4}\hbar^2$

Q.No:10 GATE-2017

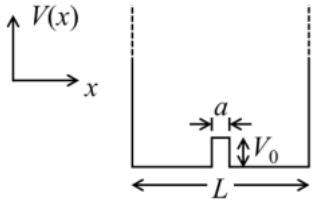
A one dimensional simple harmonic oscillator with Hamiltonian $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$ is subjected to a small perturbation, $H_1 = \alpha x + \beta x^3 + \gamma x^4$. The first order correction to the ground state energy is dependent on
(A) only β
(B) α and γ
(C) α and β
(D) only γ

Q.No:11 GATE-2018

The intrinsic/permanent electric dipole moment in the ground state of hydrogen atom is (a_0 is the Bohr radius)
(A) $-3ea_0$
(B) zero
(C) ea_0
(D) $3ea_0$

Q.No:12 GATE-2018

The ground state energy of a particle of mass m in an infinite potential well is E_0. It changes to $E_0(1 + \alpha \times 10^{-3})$, when there is a small potential bump of height $V_0 = \frac{\pi^2\hbar^2}{50mL^2}$ and width $a = L/100$, as shown in the figure. The value of α is _____ (up to two decimal places).



Q.No:13 GATE-2019

An electric field $\vec{E} = E_0\hat{z}$ is applied to a Hydrogen atom in $n = 2$ excited state. Ignoring spin, the $n = 2$ state is fourfold degenerate, which in the $|l, m\rangle$ basis are given by $|0, 0\rangle, |1, 1\rangle, |1, 0\rangle$ and $|1, -1\rangle$. If H' is the interaction Hamiltonian corresponding to the applied electric field, which of the following matrix elements is nonzero?
(A) $\langle 0, 0|H'|0, 0\rangle$
(B) $\langle 0, 0|H'|1, 1\rangle$
(C) $\langle 0, 0|H'|1, 0\rangle$
(D) $\langle 0, 0|H'|1, -1\rangle$

Q.No:14 GATE-2019

The spin-orbit interaction term of an electron moving in a central field is written as $f(r)\vec{l} \cdot \vec{s}$, where r is the radial distance of the electron from the origin. If an electron moves inside a uniformly charged sphere, then
(A) $f(r) =$ constant
(B) $f(r) \propto r^{-1}$
(C) $f(r) \propto r^{-2}$
(D) $f(r) \propto r^{-3}$

Q.No:15 GATE-2019

The Hamiltonian of a system is $H = \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & -1 \end{pmatrix}$ with $\varepsilon \ll 1$. The fourth order contribution to the ground state energy of H is $\gamma\varepsilon^4$. The value of γ (rounded off to three decimal places) is _____.

Q.No:16 GATE-2020

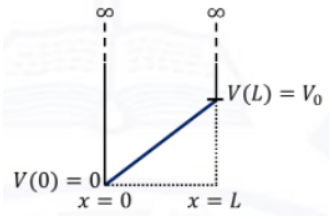
Consider the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}'$ where $\hat{H}_0 = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$ and $\hat{H}'$ is the time independent perturbation given by $\hat{H}' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}$, where $k > 0$. If, the maximum energy eigenvalue of $\hat{H}$ is $3eV$ corresponding to $E = 2eV$, the value of k (rounded off to three decimal places) in eV is _____.

Q.No:17 GATE-2021

Consider a particle in a one-dimensional infinite potential well with its walls at $x = 0$ and $x = L$. The system is

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perturbed as shown in the figure.



The first order correction to the energy eigenvalue is
(A) $\frac{V_0}{4}$
(B) $\frac{V_0}{3}$
(C) $\frac{V_0}{2}$
(D) $\frac{V_0}{5}$

Q.No:18 GATE-2021

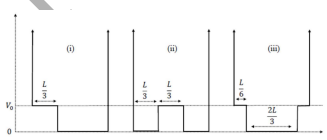
For a two-nucleon system in spin singlet state, the spin is represented through the Pauli matrices σ_1, σ_2 for particles 1 and 2, respectively. The value of $(\sigma_1 \cdot \sigma_2)$ (in integer) is _____.

Q.No:19 GATE-2021

The spin $\vec{S}$ and orbital angular momentum $\vec{L}$ of an atom precess about $\vec{J}$, the total angular momentum. $\vec{J}$ precesses about an axis fixed by a magnetic field $\vec{B}_1 = 2B_0\hat{z}$, where B_0 is a constant. Now the magnetic field is changed to $\vec{B}_2 = B_0(\hat{x} + \sqrt{2}\hat{y} + \hat{z})$. Given the orbital angular momentum quantum number $l = 2$ and spin quantum number $s = 1/2$, θ is the angle between $\vec{B}_1$ and $\vec{J}$ for the largest possible values of total angular quantum number j and its z-component j_z. The value of θ (in degree, rounded off to the nearest integer) is _____.

Q.No:20 GATE-2022

Consider a particle in three different boxes of width L. The potential inside the boxes vary as shown in figures (i), (ii) and (iii) with $V_0 \ll \frac{\hbar^2\pi^2}{2mL^2}$. The corresponding ground-state energies of the particle are E_1, E_2 and E_3, respectively. Then



(a) $E_2 > E_1 > E_3$
(b) $E_3 > E_1 > E_2$
(c) $E_2 > E_3 > E_1$
(d) $E_3 > E_2 > E_1$

Q.No:21 GATE-2022

A particle of mass m in the x-y plane is confined in an infinite two-dimensional well with vertices at $(0,0), (0,L), (L,L), (L,0)$. The eigenfunctions of this particle are $\psi_{n_x,n_y} = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$. If perturbation of the form $V = Cxy$, where C is a real constant, is applied, then which of the following statements are correct for the first excited state?

(a) The unperturbed energy is $\frac{3\pi^2\hbar^2}{2mL^2}$
(b) The unperturbed energy is $\frac{5\pi^2\hbar^2}{2mL^2}$
(c) First order energy shift due to the applied perturbation is zero
(d) The shift (δ) in energy due to the applied perturbation is determined by an equation of the form $\begin{vmatrix} a-\delta & b \\ b & a-\delta \end{vmatrix} = 0$, where a and b are real, non-zero constants

Q.No:22 GATE-2024

The non-relativistic Hamiltonian for a single electron atom is

$$H_0 = \frac{p^2}{2m} - V(r)$$

where $V(r)$ is the Coulomb potential and m is the mass of the electron. Considering the spin-orbit interaction term

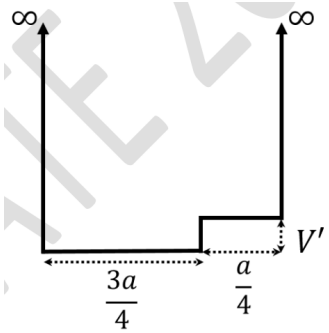
$$H' = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\vec{L}\cdot\vec{S}$$

added to H_0, which of the following statement is/are true?

1) H' commutes with L^2
2) H' commutes with L_z and S_z
3) For a given value of principal quantum number n and orbital angular momentum quantum number l, there are $2(2l+1)$ degenerate eigenstates of H_0
4) H_0, L^2, S^2, L_z, and S_z have a set of simultaneous eigenstates

Q.No:23 GATE-2024

A particle of mass m in an infinite potential well of width a is subjected to a perturbation, $V' = \frac{h^2}{40ma^2}$ as shown in figure,



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where h is Planck's constant.

The first order energy shift of the fourth energy eigenstate due to this perturbation is

$$\frac{h^2}{Nma^2}$$

The value of N is_____ (in integer).

Q.No:24 GATE-2025

A two–level quantum system has energy eigenvalues E_1 and E_2. A perturbing potential $H' = \lambda\Delta\sigma_x$ is introduced, where Δ is a constant having dimensions of energy, λ is a small dimensionless parameter, and

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The magnitudes of the first and the second order corrections to E_1 due to H', respectively, are

(1) 0 and $\frac{\lambda^2\Delta^2}{|E_1-E_2|}$
(2) $\frac{|\lambda\Delta|}{2}$ and $\frac{\lambda^2\Delta^2}{|E_1-E_2|}$
(3) $|\lambda\Delta|$ and $\frac{\lambda^2\Delta^2}{|E_1-E_2|}$
(4) 0 and $\frac{1}{2}\frac{\lambda^2\Delta^2}{|E_1-E_2|}$

Q.No:25 GATE-2026

If the perturbation

$$V = \lambda x^3$$

is added to the Hamiltonian of a one-dimensional harmonic oscillator, the matrix element $\langle m|V|0\rangle$ is/are nonzero for which of the following states? Here, the eigenstates of the harmonic oscillator are denoted by $|n\rangle$.

(A) $|m = 3\rangle$

(B) $|m = 1\rangle$

(C) $|m = 2\rangle$

(D) $|m = 5\rangle$

Q.No:26 GATE-2026

The Hamiltonian for a quantum particle of mass m is given below, where $\omega < \Omega$. The Schrödinger equation for this system can be solved exactly using the orthogonal transformations:

$$x = \frac{x_1 - x_2}{\sqrt{2}} \quad \text{and} \quad y = \frac{x_1 + x_2}{\sqrt{2}}.$$

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] + \frac{1}{2}m\Omega^2(x^2 + y^2) + m\omega^2 xy.$$

The ground state energy of this system is

(A) $\frac{\hbar}{2}\left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2}\right]$

(B) $\hbar\left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2}\right]$

(C) $\frac{\hbar}{2}\left[\sqrt{\Omega^2 + \Omega\omega} + \sqrt{\Omega^2 - \Omega\omega}\right]$

(D) $\hbar\left[\sqrt{\Omega^2 + \Omega\omega} - \sqrt{\Omega^2 - \Omega\omega}\right]$

Q.No:27 GATE-2026

The Hamiltonian of two interacting spin-$\frac{1}{2}$ particles is

$$H = \frac{A}{\hbar^2}\,\vec{S}_1 \cdot \vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spin angular momenta of particles 1 and 2, respectively. Here, $A = 10.56\,\text{eV}$. The energy in eV required to induce an excitation from the ground state to the excited state (rounded off to two decimal places) is _____

Q.No: 1 CSIR Dec-2014

The Hamiltonian H_0 for a three-state quantum system is given by the matrix $H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. When perturbed by $H' = \epsilon\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ where $\epsilon \ll 1$, the resulting shift in the energy eigenvalue $E_0 = 2$ is

(1) $\epsilon, -2\epsilon$
(2) $-\epsilon, 2\epsilon$
(3) $\pm\epsilon$
(4) $\pm 2\epsilon$

Q.No:2 CSIR Dec-2014

The effective spin-spin interaction between the electron spin $\vec{s}_e$ and the proton spin $\vec{s}_p$ in the ground state of the Hydrogen atom is given by $H' = a\vec{s}_e \cdot \vec{s}_p$. As a result of this interaction, the energy levels split by an amount

(1) $\frac{1}{2}a\hbar^2$
(2) $2a\hbar^2$
(3) $a\hbar^2$
(4) $\frac{3}{2}a\hbar^2$

Q.No:3 CSIR Dec-2015

A hydrogen atom is subjected to the perturbation $V_{pert}(\mathbf{r}) = \epsilon\cos(2r/a_0)$, where a_0 is the Bohr radius. The change in the ground state energy to first order in ϵ is

(1) $\epsilon/4$
(2) $\epsilon/2$
(3) $-\epsilon/2$
(4) $-\epsilon/4$

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Q.No:4 CSIR June-2016

Consider a particle of mass m in a potential $V(x) = \frac{1}{2}m\omega^2x^2 + g\cos kx$. The change in the ground state energy, compared to the simple harmonic potential $\frac{1}{2}m\omega^2x^2$, to first order in g is

(1) $g\exp\left(-\frac{k^2\hbar}{2m\omega}\right)$
(2) $g\exp\left(\frac{k^2\hbar}{2m\omega}\right)$
(3) $g\exp\left(-\frac{2k^2\hbar}{m\omega}\right)$
(4) $g\exp\left(-\frac{k^2\hbar}{4m\omega}\right)$

Q.No: 5 CSIR June-2017

The Coulomb potential $V(r) = -e^2/r$ of a hydrogen atom is perturbed by adding $H' = bx^2$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is (The ground state wave-function is $\psi_0 = \frac{1}{\sqrt{\pi a_0^2}}e^{-r/a_0}$.)

(1) $2ba_0^2$
(2) ba_0^2
(3) $ba_0^2/2$
(4) $\sqrt{2}ba_0^2$

Q.No: 6 CSIR Dec-2017

Consider a one-dimensional infinite square well

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a, \\ \infty & \text{otherwise} \end{cases}$$

If a perturbation

$$\Delta V(x) = \begin{cases} V_0 & \text{for } 0 < x < a/3, \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state, to first order in ΔV, is nearest to

(1) V_0
(2) $0.16V_0$
(3) $0.2V_0$
(4) $0.33V_0$

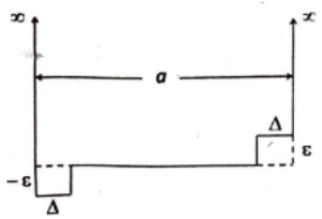
Q.No: 7 CSIR June-2018

A particle of mass m is constrained to move in a circular ring of radius R. When a perturbation $V' = \frac{a}{R^2}\cos^2\phi$ (where a is a real constant) is added, the shift in energy of the ground state, to first order in a, is

(1) a/R^2
(2) $2a/R^2$
(3) $a/(2R^2)$
(4) $a/(\pi R^2)$

Q.No: 8 CSIR June-2019

The infinite square-well potential of a particle in a box of size a is modified as shown in the figure below (assume $\Delta \ll a$)



The energy of the ground state, compared to the ground state energy before the perturbation was added

(1) increases by a term of order ε
(2) decreases by a term of order ε
(3) increases by a term of order ε^2
(4) decreases by a term of order ε^2

Q.No: 9 CSIR Dec-2019

The Hamiltonian of two interacting particles, one with spin 1 and the other with spin $\frac{1}{2}$, is given by $H = A\mathbf{S}_1 \cdot \mathbf{S}_2 + B(S_{1x} + S_{2x})$, where $\mathbf{S}_1$ and $\mathbf{S}_2$ denote the spin operators of the first and second particles, respectively, and A and B are positive constants. The largest eigenvalue of this Hamiltonian is

(1) $\frac{1}{2}(A\hbar^2 + 3B\hbar)$
(2) $3A\hbar^2 + B\hbar$
(3) $\frac{1}{2}(3A\hbar^2 + B\hbar)$
(4) $A\hbar^2 + 3B\hbar$

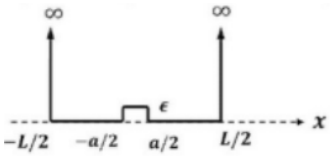
Q.No: 10 Assam CSIR Dec-2019

Consider the degenerate first excited states $n = 2, l = 1$ of the hydrogen atom. If its Hamiltonian is modified by adding $H_1 = \frac{\alpha}{\hbar}L_x$, where L_x is the x-component of the orbital angular momentum, the changes in the energies of the states labelled by $m = 0, +1$ and -1 are, respectively,

(1) $0, \alpha$ and $-\alpha$
(2) $\alpha, 2\alpha$ and 0
(3) $\alpha, 0$ and $-\alpha$
(4) $0, 2\alpha$ and -2α

Q.No: 11 Assam CSIR Dec-2019

A quantum particle in a one-dimensional box of length L is perturbed by a barrier of height ϵ and length a, as shown in the figure.



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Assume that ϵ is much smaller than the ground state energy of the particle in the unperturbed box and $a \ll L$. The changes of the two lowest energy eigenvalues, to first order in ϵ and first order in a, are, respectively,

(1) $\frac{2\epsilon a}{L}$ and $-\frac{2\epsilon a}{L}$
(2) 0 and $\frac{2\epsilon a}{L}$
(3) $-\frac{2\epsilon a}{L}$ and 0
(4) $\frac{2\epsilon a}{L}$ and 0

Q.No: 12 CSIR June-2020

Two coupled oscillators in a potential $V(x,y) = \frac{1}{2}kx^2 + 2xy + \frac{1}{2}ky^2$ $(k > 2)$ can be decoupled into two independent harmonic oscillators (coordinates: x', y') by means of an appropriate transformation $\begin{pmatrix} x' \\ y' \end{pmatrix} = S \begin{pmatrix} x \\ y \end{pmatrix}$. The transformation matrix S is

(a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & 1 \\ 1 & -\frac{1}{\sqrt{2}} \end{pmatrix}$
(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$
(c) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$
(d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Q.No: 13 CSIR June-2020

If we take the nuclear spin I into account, the total angular momentum is $\vec{F} = \vec{L} + \vec{S} + \vec{I}$, where $\vec{L}$ and $\vec{S}$ are the orbital and spin angular momenta of the electron. The Hamiltonian of the hydrogen atom is corrected by the additional interaction $\lambda \vec{I} \cdot (\vec{L} + \vec{S})$, where $\lambda > 0$ is a constant. The total angular momentum quantum number F of the p-orbital state with the lowest energy is

(a) 0
(b) 1
(c) 1/2
(d) 3/2

Q.No: 14 CSIR Sep-2022

To first order in perturbation theory, the energy of the ground state of the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{\hbar\omega}{\sqrt{512}} \exp\left[-\frac{m\omega}{\hbar}x^2\right]$$

(1) $\frac{15}{32}\hbar\omega$
(2) $\frac{17}{32}\hbar\omega$
(3) $\frac{19}{32}\hbar\omega$
(4) $\frac{21}{32}\hbar\omega$

Q.No: 15 CSIR June 2023

A particle in one dimension is in an infinite potential well between $\frac{-L}{2} \le x \le \frac{L}{2}$. For a perturbation $\epsilon \cos(\frac{\pi x}{L})$, where ϵ is a small constant, the change in the energy of the ground state, to first order in ϵ, is

1) $\frac{5\epsilon}{\pi}$
2) $\frac{10\epsilon}{3\pi}$
3) $\frac{8\epsilon}{3\pi}$
4) $\frac{4\epsilon}{\pi}$

Q.No: 16 CSIR June 2024

An integral is given by

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \exp\left[-(x^2 + y^2 + 2axy)\right],$$

where a is a real parameter. The full range of values of a for which the integral is finite, is:

1) $-\infty < a < \infty$
2) $-2 < a < 2$
3) $-1 < a < 1$
4) $-1 \le a \le 1$

Q.No: 17 CSIR Dec 2024

Consider a particle in a one–dimensional infinite potential well between $0 \le x \le L$. If a small perturbation,

$$V(x) = \lambda \cos\left(\frac{\pi x}{L}\right),$$

(where $\lambda \ll 1$) is applied, the first–order energy correction to the ground state is

1) λ
2) 0
3) $-\lambda$
4) 2λ

Q.No: 18 CSIR June 2025

The ground state wavefunction for the hydrogen atom is

$$\psi_0 = \sqrt{\frac{1}{\pi a_0^3}}\, e^{-r/a_0}$$

where a_0 is the Bohr radius. Considering an additional potential H' as a perturbation to the hydrogen atom Hamiltonian, given by

$$H' = \left(\frac{e^2}{4\pi\epsilon_0}\right)\left[\frac{1}{r} - \frac{1}{R}\right]; 0 < r < R,$$

$H' = 0$; otherwise. where R is the radius of the proton, $R \ll a_0$. The shift in the ground state energy due to H' is

1) $\left(\frac{e^2}{4\pi\epsilon_0 a_0}\right)\frac{4R^2}{3a_0^2}$
2) $\left(\frac{e^2}{4\pi\epsilon_0 a_0}\right)\frac{R}{a_0}$

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3) $-\left(\frac{e^2}{4\pi\epsilon_0 a_0}\right)\frac{2R^2}{a_0^2}$

4) $\left(\frac{e^2}{4\pi\epsilon_0 a_0}\right)\frac{2R^2}{3a_0^2}$

Q.No: 19 CSIR Dec 2025

Consider the one-dimensional motion of a particle of positive charge q confined to an infinite potential well

$$V(x) = \begin{cases} 0, & 0 \le x \le \pi, \\ \infty, & \text{otherwise}, \end{cases}$$

which is subjected to a perturbing electric field

$$\vec{E} = E_0\hat{x}.$$

The shift in the ground state energy, to the first order in q, is

1. $\frac{q\pi E_0}{2}$
2. $-\frac{q\pi E_0}{2}$
3. $q\pi E_0$
4. $-q\pi E_0$

Q.No: 20 CSIR June 2026

The Hamiltonian of a particle of mass m, moving in a circle of a fixed radius R, is given by

$$H_0 = \frac{L_z^2}{2mR^2},$$

where

$$L_z = -i\hbar\frac{d}{d\phi}$$

and ϕ is the azimuthal angle. This particle is perturbed by

$$H' = V\cos(2\phi)$$

with

$$V \ll \frac{\hbar^2}{2mR^2}.$$

Within the first-order degenerate perturbation theory, the magnitude of the energy splitting of the first excited state is:

1. $\frac{V}{2}$
2. V
3. $2V$
4. $\frac{V}{4}$

Q.No: 21 CSIR June 2026

Consider a particle of mass m in an infinite potential well with walls at $x = 0$ and $x = 2L$. The particle is perturbed by a weak perturbation

$$H' = V_0 L\delta\left(x - \frac{3L}{2}\right).$$

The first-order correction to its first excited state energy is:

1. $2V_0$
2. V_0
3. $\frac{V_0}{2}$
4. 0

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Question No.	Entrance with year	
1	GATE-2012	Option (A)
2	GATE-2013	Option (C)
3	GATE-2013	Option (C)
4	GATE-2014	Option (A)
5	GATE-2014	Option (C)
6	GATE-2015	Option (D)
7	GATE-2015	-0.3 eV
8	GATE-2016	2
9	GATE-2016	Option (B)
10	GATE-2017	Option (D)
11	GATE-2018	Option (B)
12	GATE-2018	0.78-0.82
13	GATE-2019	Option (C)
14	GATE-2019	Option (A)
15	GATE-2019	0.125
16	GATE-2020	0.706-0.708
17	GATE-2021	Option (C)
18	GATE-2021	-3
19	GATE-2021	27-93
20	GATE-2022	Option (a)
21	GATE-2022	Options (b), (d)
22	GATE-2024	Options 1, 3, 4
23	GATE-2024	160
24	GATE-2025	Option (1)
25	GATE-2026	Option (A),(B)
26	GATE-2026	Option (A)
27	GATE-2026	10.56
1	CSIR Dec-2014	Option (3)
2	CSIR Dec-2014	Option (3)
3	CSIR Dec-2015	Option (4)
4	CSIR June-2016	Option (4)
5	CSIR June-2017	Option (2)
6	CSIR Dec-2017	-
7	CSIR June-2018	Option (3)
8	CSIR June-2019	Option (4)
9	CSIR Dec-2019	Option (1)
10	Assam CSIR Dec-2019	Option (1)
11	Assam CSIR Dec-2019	Option (4)
12	CSIR June-2020	Option (b)
13	CSIR June-2020	Option (b)
14	CSIR Sep-2022	Option (2)
15	CSIR June-2023	Option 3
16	CSIR June-2024	Option 3
17	CSIR Dec-2024	Option 2
18	CSIR June-2025	Option 4
19	CSIR Dec-2025	Option 2
20	CSIR June-2026	Option 2
21	CSIR June-2026	Option 2

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