Variational WKB and Scattering GATE, CSIR, JEST TIFR

https://snboseplc.com/variational-wkb-and-scattering-gate-csir-jest-tifr/

GATE Questions

Q.No:1 GATE-2016

The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresponding Born approximation is known as the first Born approximation. Such an approximation is valid for

(A) large incident energies and weak scattering potentials.

(B) large incident energies and strong scattering potentials.

(C) small incident energies and weak scattering potentials.

(D) small incident energies and strong scattering potentials.

Q.No:2 GATE-2016

Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90° and the magnitude of the incident wave vector is equal to $\sqrt{2\pi}\ fm^{-1}$ then the total scattering cross section in units of fm^2 is ---------.

Q.No:3 GATE-2016

Protons and α-particles of equal initial momenta are scattered off a gold foil in a Rutherford scattering experiment. The scattering cross sections for proton on gold and α-particle on gold are σ_p and σ_α respectively. The ratio σ_α/σ_p is ---------.

Q.No:4 GATE-2021

Consider the potential $U(r)$ defined as

$$U(r) = -U_0 \frac{e^{-\alpha r}}{r}$$

where α and U_0 are real constants of appropriate dimensions. According to the first Born approximation, the elastic scattering amplitude calculated with $U(r)$ for a (wave-vector) momentum transfer q and $\alpha \to 0$, is proportional to

(*Useful integral:* $\int_0^\infty \sin(qr) e^{-\alpha r} dr = \frac{q}{\alpha^2+q^2}$)

(A) q^{-2}

(B) q^{-1}

(C) q

(D) q^2

Q.No:5 GATE-2021

Following trial wavefunctions

$$\phi_1 = e^{-Z'(r_1+r_2)}$$

and

$$\phi_2 = e^{-Z'(r_1+r_2)}(1 + g|\vec{r_1} - \vec{r_2}|)$$

are used to get a variational estimate of the ground state energy of the helium atom. Z' and g are the variational parameters, $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the electrons. Let E_0 be the exact ground state energy of the helium atom. E_1 and E_2 are the variational estimates of the ground state energy of the helium atom corresponding to ϕ_1 and ϕ_2, respectively. Which one of the following options is true?

(A) $E_1 \leq E_0,\ E_2 \leq E_0,\ E_1 \geq E_2$

(B) $E_1 \geq E_0,\ E_2 \leq E_0,\ E_1 \geq E_2$

(C) $E_1 \leq E_0,\ E_2 \geq E_0,\ E_1 \leq E_2$

(D) $E_1 \geq E_0,\ E_2 \geq E_0,\ E_1 \geq E_2$

Q.No:6 GATE-2025

A particle is scattered from a potential $V(\vec{r}) = g\,\delta^3(\vec{r})$, where g is a positive constant. Using the first Born approximation, the angular (θ, ϕ) dependence of the differential scattering cross section $\frac{d\sigma}{d\Omega}$ is

a) Independent of θ but dependent on ϕ

b) Dependent on θ but independent of ϕ

c) Dependent on both θ and ϕ

d) Independent of both θ and ϕ

Q.No:7 GATE-2026

An α particle moves towards a fixed nucleus carrying charge Ze, with initial speed v_0 and impact parameter b. Starting from a large distance from the nucleus, its distance of closest approach is r_m and its speed there is v_m. Then which of the following options is/are correct?

$$\left(k = \frac{1}{4\pi\epsilon_0} \text{ and } r_0 = \frac{kZe^2}{mv_0^2}\right)$$

(A) $v_0 b = v_m r_m$

(B) $v_0 b = 2 v_m r_0$

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(C) For $\frac{b}{r_0} \ll 1$,$r_m = 4r_0 + \frac{b^2}{2r_0}$ ignoring higher order corrections in $\frac{b}{r_0}$

(D) For $\frac{b}{r_0} \ll 1$,$r_m = 4r_0 + \frac{b^2}{8r_0}$ ignoring higher order corrections in $\frac{b}{r_0}$

CSIR Questions

Q.No: 1 CSIR Dec-2014

The ground state energy of the attractive delta functional potential

$$V(x) = -b\delta(x),$$

where $b > 0$, calculated with the variational trial function

$$\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for } -a < x < a, \\ 0, & \text{otherwise,} \end{cases}$$

is

(1) $-\frac{mb^2}{\pi^2\hbar^2}$

(2) $-\frac{2mb^2}{\pi^2\hbar^2}$

(3) $-\frac{mb^2}{2\pi^2\hbar^2}$

(4) $-\frac{mb^2}{4\pi^2\hbar^2}$

Q.No: 2 CSIR Dec-2014

Consider a particle of mass m in the potential $V(x) = a|x|$, $a > 0$. The energy eigen-values E_n $(n = 0, 1, 2, \cdots)$, in the WKB approximation, are

(1) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{1/3}$

(2) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$

(3) $\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)$

(4) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{4/3}$

Q.No: 3 CSIR June-2015

The differential cross-section for scattering by a target is given by $\frac{d\sigma}{d\Omega}(\theta,\varphi) = a^2 + b^2\cos^2\theta$. If N is the flux of the incoming particles, the number of particles scattered per unit time is.

(1) $\frac{4\pi}{3}N(a^2+b^2)$

(2) $4\pi N\left(a^2+\frac{1}{6}b^2\right)$

(3) $4\pi N\left(\frac{1}{2}a^2+\frac{1}{3}b^2\right)$

(4) $4\pi N\left(a^2+\frac{1}{3}b^2\right)$

Q.No: 4 CSIR June-2015

A particle of energy E scatters off a repulsive spherical potential where V_0, and a are positive constants. In the low energy limit, the total scattering cross section is $\sigma = 4\pi a^2\left(\frac{1}{ka}\tanh ka - 1\right)^2$, where $k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0$. In the limit $V_0 \to \infty$ the ratio of σ to the classical scattering cross-section off a sphere of radius a is

(1) 4

(2) 3

(3) 1

(4) $\frac{1}{2}$

Q.No: 5 CSIR Dec-2015

In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by

(1) $\sqrt{\frac{hc}{E}}$

(2) $\frac{hc}{E^{3/2}}$

(3) $\left(\frac{hc}{E}\right)^2$

(4) $\frac{hc}{E}$

Q.No: 6 CSIR Dec-2015

The ground state energy of a particle in the potential $V(x) = g|x|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\frac{c}{a^5}}(a^2 - x^2), & x < |a| \\ 0, & x \geq |a| \end{cases}$$

(where g and c are constants) is

(1) $\frac{15}{16}\left(\frac{\hbar^2g^2}{m}\right)^{1/3}$

(2) $\frac{5}{6}\left(\frac{\hbar^2g^2}{m}\right)^{1/3}$

(3) $\frac{3}{4}\left(\frac{\hbar^2g^2}{m}\right)^{1/3}$

(4) $\frac{7}{8}\left(\frac{\hbar^2g^2}{m}\right)^{1/3}$

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Q.No: 7 CSIR June-2016

The energy levels for a particle of mass m in the potential $V(x) = \alpha|x|$, determined in the WKB approximation

$$\sqrt{2m}\int_a^b \sqrt{E - V(x)}dx = \left(n + \frac{1}{2}\right)\hbar\pi,$$

(where a, b are the turning points and $n = 0, 1, 2\cdots$), are

(1) $E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(2) $E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(3) $E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(4) $E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

Q.No: 8 CSIR June-2016

The ground state energy of a particle of mass m in the potential $V(x) = \frac{\hbar^2\beta}{6m}x^4$, estimated using the normalized trial wavefunction $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$, is [Use $\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx x^2 e^{-\alpha x^2} = \frac{1}{2\alpha}$ and $\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx x^4 e^{-\alpha x^2} = \frac{3}{4\alpha^2}$].

(1) $\frac{3}{2m}\hbar^2\beta^{1/3}$

(2) $\frac{8}{3m}\hbar^2\beta^{1/3}$

(3) $\frac{2}{3m}\hbar^2\beta^{1/3}$

(4) $\frac{3}{8m}\hbar^2\beta^{1/3}$

Q.No: 9 CSIR Dec-2016

A particle is scattered by a central potential $V(r) = V_0 r e^{-\mu r}$, where V_0 and μ are positive constants. If the momentum transfer $\vec{q}$ is such that $q = |\vec{q}| \gg \mu$, the scattering cross-section in the Born approximation, as $q \to \infty$, depends on q as [You may use $\int x^n e^{ax} dx = \frac{d^n}{da^n}\int e^{ax} dx$]

(1) q^{-8}

(2) q^{-2}

(3) q^2

(4) q^6

Q.No: 10 CSIR June-2017

Consider the potential

$$V(\vec{r}) = \sum_i V_0 a^3 \delta^3(\vec{r} - \vec{r}_i)$$

where $\vec{r}_i$ are the position vectors of the vertices of a cube of length a centered at the origin and V_0 is a constant. If $V_0 a^2 \ll \frac{\hbar^2}{m}$, the total scattering cross-section, in the low-energy limit, is

(1) $16a^2\left(\frac{mV_0a^2}{\hbar^2}\right)$

(2) $\frac{16a^2}{\pi^2}\left(\frac{mV_0a^2}{\hbar^2}\right)^2$

(3) $\frac{64a^2}{\pi}\left(\frac{mV_0a^2}{\hbar^2}\right)^2$

(4) $\frac{64a^2}{\pi^2}\left(\frac{mV_0a^2}{\hbar^2}\right)$

Q.No: 11 CSIR June-2017

Using the trial function

$$\psi(x) = \begin{cases} A(a^2 - x^2), & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the ground state energy of a one-dimensional harmonic oscillator is

(1) $\hbar\omega$

(2) $\sqrt{\frac{5}{14}}\hbar\omega$

(3) $\frac{1}{2}\hbar\omega$

(4) $\sqrt{\frac{5}{7}}\hbar\omega$

Q.No: 12 CSIR Dec-2017

A phase shift of 30° is observed when a beam of particles of energy $0.1MeV$ is scattered by a target. When the beam energy is changed, the observed phase shift is 60°. Assuming that only s-wave scattering is relevant and that the cross-section does not change with energy, the beam energy is

(1) $0.4MeV$

(2) $0.3MeV$

(3) $0.2MeV$

(4) $0.15MeV$

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Q.No: 13 CSIR Dec-2017

The energy eigenvalues E_n of a quantum system in the potential $V = cx^6$ (where $c > 0$ is a constant), for large values of the quantum number n, varies as

(1) $n^{4/3}$

(2) $n^{3/2}$

(3) $n^{5/4}$

(4) $n^{6/5}$

Q.No:14 CSIR June-2018

The n-th energy eigenvalue E_n of a one-dimensional Hamiltonian $H = \frac{p^2}{2m} + \lambda x^4$ (where $\lambda > 0$ is a constant) in the WKB approximation, is proportional to

(1) $(n + \frac{1}{2})^{4/3}\lambda^{1/3}$

(2) $(n + \frac{1}{2})^{4/3}\lambda^{2/3}$

(3) $(n + \frac{1}{2})^{5/3}\lambda^{1/3}$

(4) $(n + \frac{1}{2})^{5/3}\lambda^{2/3}$

Q.No:15 CSIR June-2018

The differential scattering cross-section $\frac{d\sigma}{d\Omega}$ for the central potential $V(r) = \frac{\beta}{r}e^{-\mu r}$, where β and μ are positive constants, is calculated in the first Born approximation. Its dependence on the scattering angle θ is proportional to (A is a constant below.)

(1) $(A^2 + \sin^2\frac{\theta}{2})$

(2) $(A^2 + \sin^2\frac{\theta}{2})^{-1}$

(3) $(A^2 + \sin^2\frac{\theta}{2})^{-2}$

(4) $(A^2 + \sin^2\frac{\theta}{2})^{2}$

Q.No:16 CSIR June-2019

The Hamiltonian of a quantum particle of mass m is $H = \frac{p^2}{2m} + \alpha|x|^r$, where α and r are positive constants. The energy E_n of the n^{th} level, for large n, depends on n as

(1) n^{2r}

(2) n^{r+2}

(3) $n^{1/(r+2)}$

(4) $n^{2r/(r+2)}$

Q.No:17 CSIR June-2019

In the partial wave expansion, the differential scattering cross-section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \left|\sum_l (2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta)\right|^2$$

where θ is the scattering angle. For a certain neutron-nucleus scattering, it is found that the two lowest phase shifts δ_0 and δ_1 corresponding to s-wave and p-wave, respectively, satisfy $\delta_1 \approx \delta_0/2$. Assuming that the other phase shifts are negligibly small, the differential cross-section reaches its minimum for $\cos\theta$ equal to

(1) 0

(2) ± 1

(3) $-\frac{2}{3}\cos^2\delta_1$

(4) $\frac{1}{3}\cos^2\delta_1$

Q.No: 18 CSIR June-2019

The elastic scattering of a charged particle of mass m off an atom can be approximated by the potential $V(r) = \frac{\alpha}{r}e^{-r/R}$, where α and R are positive constants. If the wave number of the incoming particle is k and the scattering angle is 2θ, the differential cross-section in the Born approximation is

(1) $\frac{m^2\alpha^2R^4}{4\hbar^4(1+k^2R^2\sin^2\theta)}$

(2) $\frac{m^2\alpha^2R^4}{\hbar^4(2k^2R^2+\sin^2\theta)^2}$

(3) $\frac{2m^2\alpha^2R^4}{\hbar^4(2k^2R^2+\sin^2 2\theta)}$

(4) $\frac{4m^2\alpha^2R^4}{4\hbar^4(1+4k^2R^2\sin^2\theta)^2}$

Q.No: 19 CSIR June-2020

A particle with incoming wave vector $\vec{k}$, after being scattered by the potential $V(r) = \frac{c}{r^2}$, goes out with wave vector $\vec{k'}$. The differential scattering cross-section, calculated in the first Born approximation, depends on $q = |\vec{k} - \vec{k'}|$, as

(a) $1/q^2$

(b) $1/q^4$

(c) $1/q$

(d) $1/q^{3/2}$

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Q.No: 20 CSIR Feb-2022

The Hamiltonian of a particle of mass m in one-dimension is $H = \frac{1}{2m}p^2 + \lambda|x|^3$, where $\lambda > 0$ is a constant. If E_1 and E_2 respectively, denote the ground state energies of the particle for $\lambda = 1$ and $\lambda = 2$ (in appropriate units) the ratio E_1/E_2 is best approximated by

(1) 1.260

(2) 1.414

(3) 1.516

(4) 1.320

Q.No: 21 CSIR Feb-2022

In an elastic scattering process at an energy E , the phase shifts satisfy $\delta_0 \approx 30^0, \delta_1 \approx 10^0$, while the other phase shifts are zero. The polar angle at which the differential cross section peaks is closest to

(1) 20^0

(2) 10^0

(3) 0^0

(4) 30^0

Q.No: 22 CSIR June-2015

A particle of energy E scatters off a repulsive spherical potential where V_0, and a are positive constants. In the low energy limit, the total scattering cross section is $\sigma = 4\pi a^2 \left(\frac{1}{ka}\tanh ka - 1\right)^2$, where $k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0$. In the limit $V_0 \to \infty$ the ratio of σ to the classical scattering cross-section off a sphere of radius a is

(1) 4

(2) 3

(3) 1

(4) 1/2

Q.No: 23 CSIR Dec-2023

An incident plane wave with wavenumber k is scattered by a spherically symmetric soft potential. The scattering occurs only in S- and P- waves. The approximate scattering amplitude at angles $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$ are $f\left(\theta = \frac{\pi}{3}\right) \simeq \frac{1}{2k}\left(\frac{5}{2} + 3i\right)$ and $f\left(\theta = \frac{\pi}{2}\right) \simeq \frac{1}{2k}\left(1 + \frac{3i}{2}\right)$.

Then the total scattering cross-section is closest to

1) $\frac{37\pi}{4k^2}$

2) $\frac{10\pi}{k^2}$

3) $\frac{35\pi}{4k^2}$

4) $\frac{9\pi}{k^2}$

Q.No: 24 CSIR June-2023

The phase shifts of the partial waves in an elastic scattering at energy E are $\delta_0 = 12^0$, $\delta_1 = 4^0$ and $\delta_{l\geq 2} \simeq 0^0$. The best qualitative depiction of θ-dependence of the differential scattering cross-section $\frac{d\sigma}{d\cos\theta}$ is

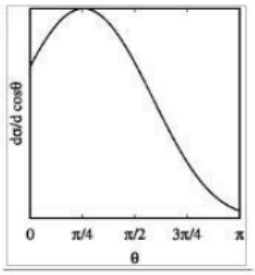


Figure 1: (a)

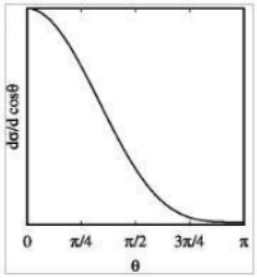


Figure 2: (b)

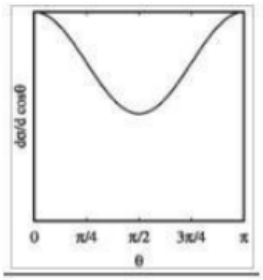


Figure 3: (c)

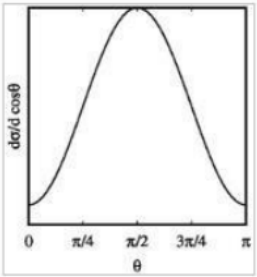


Figure 4: (d)

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Q.No: 26 CSIR June-2024

The Hamiltonian of a particle of mass m is given by

$$H = \frac{p^2}{2m} + V(x),$$

with

$$V(x) = \begin{cases} -\alpha x & \text{for } x \leq 0 \\ \beta x & \text{for } x > 0 \end{cases}$$

where α, β are positive constants. The n^{th} energy eigenvalue E_n obtained using WKB approximation is

$$E_n^{3/2} = \frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2} \pi \left(n - \frac{1}{2}\right) f(\alpha, \beta) \quad (n = 1, 2, \ldots).$$

The function $f(\alpha, \beta)$ is

1) $\frac{\sqrt{\alpha^2\beta^2}}{\sqrt{2(\alpha^2+\beta^2)}}$

2) $\frac{\alpha\beta}{\alpha+\beta}$

3) $\frac{\alpha+\beta}{4}$

4) $\frac{1}{2}\sqrt{\frac{\alpha^2+\beta^2}{2}}$

Q.No: 27 CSIR June-2024

In a scattering experiment, a beam of e^- with an energy of 420 MeV scatters off an atomic nucleus. If the first minimum of the differential cross section is observed at a scattering angle of 45°, the radius of the nucleus (in fermi) is closest to

1) 0.4

2) 8.0

3) 2.5

4) 0.8

Q.No: 28 CSIR June-2025

When a neutron of 1 keV kinetic energy impinges on a ^{12}C target, the total scattering cross section is 1000 barns. The approximate value of the phase shift δ_0 is:

1) 18°

2) 108°

3) 90°

4) 36°

Q.No: 29 CSIR June-2026

A particle of mass m with momentum $\hbar\vec{k}$ scatters elastically to momentum $\hbar(\vec{k} + \vec{q})$ from a spherical potential

$$V(\vec{r}) = \begin{cases} V_0, & r \leq R, \\ 0, & r > R. \end{cases}$$

Here,

$$q^2 = 4k^2 \sin^2\frac{\theta}{2},$$

with θ being the scattering angle. Within the first-order Born approximation and for $qR \ll 1$, the differential scattering cross-section to the leading order in R is proportional to:

1. R^2
2. R^4
3. R^6
4. R^8

Q.No: 30 CSIR June-2026

The surface of a metal of work function ϕ_0 is subjected to an electric field $\mathcal{E}$. As a result, the electric potential profile just outside the surface is given by

$$V(x) = \phi_0 - x\mathcal{E}$$

as shown in the figure. For electric field $\mathcal{E}$, the probability $p(\mathcal{E})$ of tunneling of an electron moving at the Fermi energy E_F can be found using the WKB approximation. If

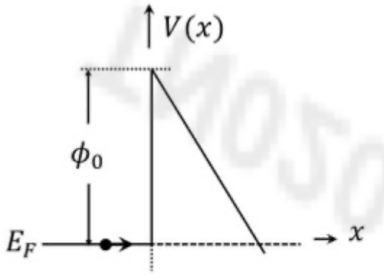
$$\lambda = \sqrt{\frac{2me\phi_0^3}{\hbar^2}},$$

then the value of the ratio

$$\frac{p(\mathcal{E} = 2\lambda)}{p(\mathcal{E} = \lambda)}$$

will be:

(Here, e and m are electron's charge and mass, respectively.)



1. 1.36
2. 2.73
3. 1.94
4. 5.13

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JEST Questions

Q.No: 1 JEST-2017

Consider a particle confined by a potential $V(x) = k|x|$, where k is a positive constant. The spectrum E_n of the system, within the WKB approximation, is proportional to

(A) $(n + \frac{1}{2})^{3/2}$.

(B) $(n + \frac{1}{2})^{2/3}$.

(C) $(n + \frac{1}{2})^{1/2}$.

(D) $(n + \frac{1}{2})^{4/3}$.

TIFR Questions

Q.No: 1 TIFR-2014

A particle P of mass m moves under the influence of a central potential, centred at the origin O, of the form

$$V(r) = -\frac{k}{3r^3}$$

where k is a positive constant. If the particle P comes in from infinity with initial velocity u and impact parameter b (see figure), then the largest value of b for which the particle gets captured by the potential is

(a) $\left(\frac{3k^2}{m^2u^4}\right)^{1/6}$

(b) $\left(\frac{k}{3mu}\right)^{1/3}$

(c) $\left(\frac{2k^2}{m^2u^4}\right)^{1/6}$

(d) $\left(\frac{2k}{3mu}\right)^{1/3}$

Q.No: 2 TIFR-2016

In a Rutherford scattering experiment, the number N of particles scattered in a direction θ, i.e. $dN/d\theta$, as a function of the scattering angle θ (in the laboratory frame) varies as

(a) $\csc^4 \frac{\theta}{2}$

(b) $\csc^2 \frac{\theta}{2} \cot \frac{\theta}{2}$

(c) $\csc^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}$

(d) $\sec^4 \frac{\theta}{2}$

Q.No: 3 TIFR-2023

Consider a particle of mass m in a quartic potential

$$H = \frac{p^2}{2m} + ax^4$$

If we take a variational wavefunction

$$\psi(x, \lambda) = e^{-\lambda x^2}$$

with $\lambda > 0$ and try to estimate the ground state energy, the value of λ should be chosen as

[You may use the integral

$$\int_{-\infty}^{+\infty} dx(A+Bx^2+Cx^4)e^{-\lambda x^2} = A\sqrt{\frac{\pi}{\lambda}}+\frac{B}{2}\sqrt{\frac{\pi}{\lambda^3}}+\frac{3C}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

where A, B, C and $\lambda > 0$ are all constants.]

(a) $(\frac{5ma}{3\pi^2\hbar^2})^{1/3}$

(b) $(\frac{15ma}{8\hbar^2})^{1/3}$

(c) $(\frac{ma}{2\pi\hbar^2})^{1/3}$

(d) $(\frac{3ma}{4\hbar^2})^{1/3}$

Q.No: 4 TIFR-2024

The Hamiltonian for a Helium atom is given as $H = H_0 + H_1$, where

$$H_0 = \frac{(p_1^2 + p_2^2)}{2\mu} - \frac{2e^2}{4\pi\epsilon_0 r_1} - \frac{2e^2}{4\pi\epsilon_0 r_2}$$

and

$$H_1 = \frac{e^2}{4\pi\epsilon_0 r_{12}}$$

where μ is the reduced mass of the electron, r_1 and r_2 are the distance of the electrons from the nucleus, and r_{12} is the distance between the two electrons. The value of the first ionization potential of the Helium atom is 24.6 eV.

What is the correction due to H_1 to the ground state energy of the Helium atom, compared to H_0?

(a) 29.8 eV

(b) -29.8 eV

(c) 84.2 eV

(d) -2.6 eV

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Q.No: 5 TIFR-2026

Consider a quantum particle of mass m in one-dimension with a Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \lambda \hat{x}^8$$

with $\lambda > 0$. The ground state energy of the particle scales with λ as:

(a) $\propto \lambda^{1/5}$

(b) $\propto \lambda^{1/4}$

(c) $\propto \lambda^{1/3}$

(d) $\propto \lambda^{1/6}$

Question No.	Entrance with year
Q.No:1 GATE-2016	(A)
Q.No:2 GATE-2016	2
Q.No:3 GATE-2016	4
Q.No:4 GATE-2021	(A)
Q.No:5 GATE-2021	(D)
Q.No:6 GATE-2025	(d)
Q.No:7 GATE-2026	(A)
Q.No: 1 CSIR Dec-2014	(2)
Q.No: 2 CSIR Dec-2014	(2)
Q.No: 3 CSIR June-2015	(4)
Q.No: 4 CSIR June-2015	(1)
Q.No: 5 CSIR Dec-2015	(3)
Q.No: 6 CSIR Dec-2015	(1)
Q.No: 7 CSIR June-2016	(2)
Q.No: 8 CSIR June-2016	(4)
Q.No: 9 CSIR Dec-2016	(1)
Q.No: 10 CSIR June-2017	(3)
Q.No: 11 CSIR June-2017	(2)
Q.No: 12 CSIR Dec-2017	(2)
Q.No: 13 CSIR Dec-2017	(2)
Q.No:14 CSIR June-2018	(1)
Q.No:15 CSIR June-2018	(3)
Q.No:16 CSIR June-2019	(4)
Q.No:17 CSIR June-2019	(3)
Q.No: 18 CSIR June-2019	(4)
Q.No: 19 CSIR June-2020	(a)
Q.No: 20 CSIR Feb-2022	(4)
Q.No: 21 CSIR Feb-2022	(3)
Q.No: 22 CSIR June-2015	(1)
Q.No: 23 CSIR Dec-2023	(1)
Q.No: 24 CSIR June-2023	(b)
Q.No: 26 CSIR June-2024	(2)
Q.No: 27 CSIR June-2024	(3)
Q.No: 28 CSIR June-2025	(4)
Q.No: 29 CSIR June-2026	(3)
Q.No: 30 CSIR June-2026	(3)
Q.No: 1 TIFR-2014	(a)
Q.No: 2 TIFR-2016	(b)
Q.No: 3 TIFR-2023	(d)
Q.No: 4 TIFR-2024	(a)
Q.No: 5 TIFR-2026	(a)

S. N. Bose Physics Learning Centre