Classical Statistics CSIR

https://snboseplc.com/classical-statistics-gate/

GATE Questions

Q.No:1 GATE-2012

A classical gas of molecules, each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are ν_x, ν_y and ν_z. Then mean value of $(\nu_x + \nu_y)^2$ is

(A) $\frac{k_B T}{m}$

(B) $\frac{3}{2}\frac{k_B T}{m}$

(C) $\frac{1}{2}\frac{k_B T}{m}$

(D) $\frac{2k_B T}{m}$

Q.No:2 GATE-2012

Consider a system whose three energy levels are given by $0, \varepsilon$ and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta = \frac{1}{k_B T}$ is given by

(A) $1 + 2e^{-\beta\varepsilon}$

(B) $2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

(C) $(1 + e^{-\beta\varepsilon})^2$

(D) $1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

Q.No:3 GATE-2012

A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that $N/2$ spins are aligned parallel and the remaining $N/2$ spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is,

(A) $2Nk_B \ln 2$

(B) $\frac{N}{2}k_B \ln 2$

(C) $\frac{2N}{2}k_B \ln 2$

(D) $Nk_B \ln 2$

Q.No:4 GATE-2012

At a certain temperature T, the average speed of nitrogen molecules in air is found to be $400m/s$. The most probable and the root mean square speeds of the molecules are, respectively,

(A) $355m/s, 434m/s$

(B) $820m/s, 917m/s$

(C) $152m/s, 301m/s$

(D) $422m/s, 600m/s$

Q.No:5 GATE-2013

Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same

(A) pressure and temperature

(B) volume and temperature

(C) pressure and volume

(D) volume and energy

Q.No:6 GATE-2013

The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where a and E_0 are positive constants. The temperature of the system is

(A) negative for some energies

(B) increases monotonically with energy

(C) decreases monotonically with energy

(D) Zero

Q.No:7 GATE-2013

Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant)

(A) Zero

(B) Nk

(C) $\frac{1}{2}Nk$

(D) $Nk \ln(2)$

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Q.No:8 GATE-2013

Consider a gas of atoms obeying Maxwell-Boltzmann statistics. The average value of $e^{i\vec{a}.\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is,

(A) One

(B) Zero

(C) $e^{-\frac{1}{2}a^2mkT}$

(D) $e^{-\frac{3}{2}a^2mkT}$

Q.No:9 GATE-2013

There are four energy levels $E, 2E, 3E$ and $4E$ (where $E > 0$). The canonical partition function of two particles is, if these particles are two identical fermions

(A) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

(B) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(C) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$

(D) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Q.No:10 GATE-2013

There are four energy levels $E, 2E, 3E$ and $4E$ (where $E > 0$). The canonical partition function of two particles is, if these particles are two distinguishable particles

(A) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

(B) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(C) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$

(D) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Q.No:11 GATE-2014

At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillators is $\cdots\cdots\cdots\cdots .k_BT$ (k_B is the Boltzmann constant).

Q.No:12 GATE-2014

For a system of two bosons, each of which can occupy any of the two energy levels 0 and ε, the mean energy of the system at a temperature T with $\beta = \frac{1}{k_BT}$ is given by

(A) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(B) $\frac{1 + \varepsilon e^{-\beta\varepsilon}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(C) $\frac{2\varepsilon e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(D) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

Q.No:13 GATE-2015

Consider a system of N non-interacting spin-1/2 particles, each having a magnetic moment μ, is in a magnetic field $\vec{B} = B\hat{z}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

(A) $\Omega = \frac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$

(B) $\Omega = \frac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$

(C) $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\,\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$

(D) $\Omega = \frac{N!}{\left(N + \frac{E}{\mu B}\right)!}$

Q.No:14 GATE-2015

The entropy of a gas containing N particles enclosed in a volume V is given by $S = Nk_B \ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right)$, where E is the total energy, a is a constant and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by

(A) $\mu = -k_BT\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{5}{2}\right]$

(B) $\mu = -k_BT\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{3}{2}\right]$

(C) $\mu = -k_BT\left[\ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right) - \frac{5}{2}\right]$

(D) $\mu = -k_BT\left[\ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right) - \frac{3}{2}\right]$

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Q.No:15 GATE-2015

The average energy U of a one dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature T is given by

(A) $U = \frac{1}{2}\hbar\omega \coth\left(\frac{1}{2}\beta\hbar\omega\right)$

(B) $U = \frac{1}{2}\hbar\omega \sinh\left(\frac{1}{2}\beta\hbar\omega\right)$

(C) $U = \frac{1}{2}\hbar\omega \tanh\left(\frac{1}{2}\beta\hbar\omega\right)$

(D) $U = \frac{1}{2}\hbar\omega \cosh\left(\frac{1}{2}\beta\hbar\omega\right)$

Q.No:16 GATE-2016

The entropy S of a system of N spins, which may align either in the upward or in the downward direction, is given by $S = -k_B N[p \ln p + (1-p)\ln(1-p)]$. Here k_B is the Boltzmann constant. The probability of alignment in the upward direction is p. The value of p, at which the entropy is maximum, is ⋯⋯⋯⋯. (Give your answer upto one decimal place)

Q.No:17 GATE-2016

N atoms of an ideal gas are enclosed in a container of volume V. The volume of the container is changed to $4V$, while keeping the total energy constant. The change in the entropy of the gas, in units of $Nk_B \ln 2$, is ⋯⋯⋯⋯, where k_B is the Boltzmann constant.

Q.No:18 GATE-2016

A two-level system has energies zero and E. The level with zero energy is non-degenerate, while the level with energy E is triply degenerate. The mean energy of a classical particle in this system at a temperature T is

(A) $\frac{Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$

(B) $\frac{Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$

(C) $\frac{3Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$

(D) $\frac{3Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$

Q.No:19 GATE-2017

Consider N non-interacting, distinguishable particles in a two-level system at temperature T. The energies of the levels are 0 and ε, where $\varepsilon > 0$. In the high temperature limit ($k_BT \gg \varepsilon$), what is the population of particles in the level with energy ε?

(A) $\frac{N}{2}$

(B) N

(C) $\frac{N}{4}$

(D) $\frac{3N}{4}$

Q.No:20 GATE-2018

A microcanonical ensemble consists of 12 atoms with each taking either energy 0 state, or energy ϵ state. Both states are non-degenerate. If the total energy of this ensemble is 4ϵ, its entropy will be ⋯⋯⋯⋯. k_B (up to one decimal place), where k_B is the Boltzmann constant.

Q.No:21 GATE-2018

The partition function of an ensemble at a temperature T is

$$Z = \left(2\cosh\frac{\varepsilon}{k_BT}\right)^N,$$

where k_B is the Boltzmann constant. The heat capacity of this ensemble at $T = \frac{\varepsilon}{k_B}$ is XNk_B, where the value of X is ⋯⋯⋯⋯. (up to two decimal places).

Q.No:22 GATE-2018

The quantum effects in an ideal gas become important below a certain temperature T_Q when de Broglie wavelength corresponding to the root mean square thermal speed becomes equal to the inter-atomic separation. For such a gas of atoms of mass $2 \times 10^{-26}kg$ and number density $6.4 \times 10^{25}m^{-3}$, $T_Q =$ ⋯⋯⋯⋯. $\times 10^{-3}K$ (up to one decimal place).

($k_B = 1.38 \times 10^{-23}J/K, h = 6.6 \times 10^{-34}J-s$)

Q.No:23 GATE-2019

Consider a one-dimensional gas of N non-interacting particles of mass m with the Hamiltonian for a single particle given by,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2(x^2 + 2x)$$

The high temperature specific heat in units of $R = Nk_B$ (k_B is the Boltzmann constant) is

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(A) 1

(B) 1.5

(C) 2

(D) 2.5

Q.No:24 GATE-2019

Consider two systems A and B each having two distinguishable particles. In both the systems, each particle can exist in states with energies $0, 1, 2$ and 3 units with equal probability. The total energy of the combined system is 5 units. Assuming that the system A has energy 3 units and the system B has energy 2 units, the entropy of the combined system is $k_B \ln \lambda$. The value of λ is .....

Q.No:25 GATE-2021

Consider a system of three distinguishable particles, each having spin $S = 1/2$ such that $S_z = \pm 1/2$ with corresponding magnetic moments $\mu_z = \pm\mu$. When the system is placed in an external magnetic field H pointing along the z-axis, the total energy of the system is μH. Let x be the state where the first spin has $S_z = 1/2$. The probability of having the state x and the mean magnetic moment (in the $+z$ direction) of the system in state x are

(A) $\frac{1}{3}, \frac{-1}{3}\mu$

(B) $\frac{1}{3}, \frac{2}{3}\mu$

(C) $\frac{2}{3}, \frac{-2}{3}\mu$

(D) $\frac{2}{3}, \frac{1}{3}\mu$

Q.No:26 GATE-2021

Consider a single one-dimensional harmonic oscillator of angular frequency ω, in equilibrium at temperature $T = (k_B\beta)^{-1}$. The states of the harmonic oscillator are all non-degenerate having energy $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ with equal probability, where n is the quantum number. The Helmholtz free energy of the oscillator is

(A) $\frac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 - \exp\left(\beta\hbar\omega\right)\right]$

(B) $\frac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 - \exp\left(-\beta\hbar\omega\right)\right]$

(C) $\frac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 + \exp\left(-\beta\hbar\omega\right)\right]$

(D) $\beta^{-1}\ln\left[1 - \exp\left(-\beta\hbar\omega\right)\right]$

Q.No:27 GATE-2022

Consider a non-interacting gas of spin 1 particles, each with magnetic moment μ, placed in a weak magnetic field B, such that $\frac{\mu B}{k_B T} \ll 1$. The average magnetic moment of a particle is

(a) $\frac{2\mu}{3}\left(\frac{\mu B}{k_B T}\right)$

(b) $\frac{\mu}{2}\left(\frac{\mu B}{k_B T}\right)$

(c) $\frac{\mu}{3}\left(\frac{\mu B}{k_B T}\right)$

(d) $\frac{3\mu}{4}\left(\frac{\mu B}{k_B T}\right)$

Q.No:28 GATE-2017

Consider a triatomic molecule of the shape shown in the figure below in three dimensions. The heat capacity of this molecule at high temperature (temperature much higher than the vibrational and rotational energy scales of the molecule but lower than its bond dissociation energies) is:



Figure 1: Triatomic molecule.

(A) $\frac{3}{2}k_B$

(B) $3k_B$

(C) $\frac{9}{2}k_B$

(D) $6k_B$

Q.No:29 GATE-2023

A simple harmonic oscillator with an angular frequency ω is in thermal equilibrium with a reservoir at absolute temperature T, with $\omega = \frac{2k_B T}{\hbar}$. Which one of the following is the partition function of the system?

(A) $\frac{e}{e^2-1}$

(B) $\frac{e}{e^2+1}$

(C) $\frac{e}{e-1}$

(D) $\frac{e}{e+1}$

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Q.No:30 GATE-2023

Consider 6 identical, non-interacting, spin $\frac{1}{2}$ atoms arranged on a crystal lattice at absolute temperature T. The z-component of the magnetic moment of each of these atoms can be $\pm\mu_B$. If P and Q are the probabilities of the net magnetic moment of the solid being $2\mu_B$ and $6\mu_B$ respectively, what is the value of $\frac{P}{Q}$ (in integer)?

Q.No:31 GATE-2024

The Hamiltonian of a system of N particles in volume V at temperature T is

$$H = \sum_{i=1}^{2N} a_i q_i^2 + \sum_{i=1}^{2N} b_i p_i^2$$

where a_i and b_i are positive constants. The ensemble average of the Hamiltonian is aNk_BT, where k_B is the Boltzmann constant. The value of α is ..... (in integer).

Q.No:32 GATE-2025

A paramagnetic material containing paramagnetic ions with total angular momentum $J = \frac{1}{2}$ is kept at absolute temperature T. The ratio of the magnetic field required for 80% of the ions to be in the lowest energy state to that required for having 60% of the ions in the lowest energy state at the same temperature is

(A) $\frac{2\ln 2}{\ln(\frac{3}{2})}$

(B) $\frac{\ln 2}{\ln(\frac{3}{2})}$

(C) $\frac{3\ln 2}{\ln(\frac{3}{2})}$

(D) $\frac{\ln 3}{\ln(\frac{3}{2})}$

Q.No:33 GATE-2025

Consider a two-level system with energy states $+\varepsilon$ and $-\varepsilon$. The number of particles at $+\varepsilon$ level is N_+ and the number of particles at $-\varepsilon$ level is N_-. The total energy of the system is E and the total number of particles is $N = N_+ + N_-$. In the thermodynamic limit, the inverse of the absolute temperature of the system is

(Given: $\ln N! \simeq N\ln N - N$)

(A) $\frac{k_B}{2\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$

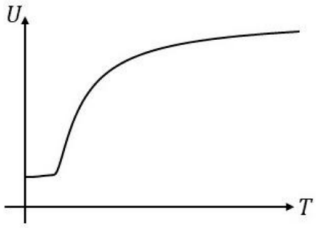
(B) $\frac{k_B}{\varepsilon}\ln N$

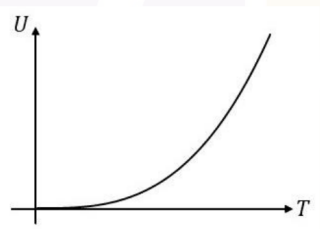
(C) $\frac{k_B}{2\varepsilon}\ln N$

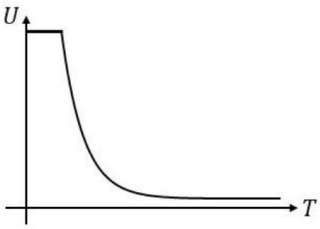
(D) $\frac{k_B}{\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$

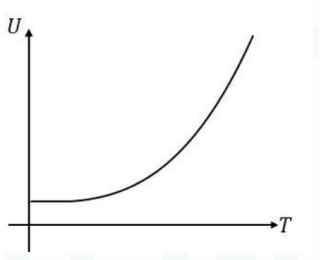
Q.No:34 GATE-2026

A gas of N classical particles that can occupy energy levels,ϵ_1 and $\epsilon_2 = \epsilon_1 + \Delta$ is in equilibrium with a reservoir at temperature T. From the schematics shown below, choose the correct dependence of the internal energy U on T.

(A)


(B)


(C)


(D)


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Question No.	**Entrance with year**	answer
1	GATE 2012	D
2	GATE 2012	C
3	GATE 2012	D
4	GATE 2012	A
5	GATE 2013	A
6	GATE 2013	A
7	GATE 2013	D
8	GATE 2013	C
9	GATE 2013	B
10	GATE 2013	C
11	GATE 2014	1.99–2.01
12	GATE 2014	A
13	GATE 2015	A
14	GATE 2015	A
15	GATE 2015	A
16	GATE 2016	0.5
17	GATE 2016	2
18	GATE 2016	D
19	GATE 2017	A
20	GATE 2018	6.1–6.3
21	GATE 2018	0.41–0.43
22	GATE 2018	78–90
23	GATE 2019	A
24	GATE 2019	12
25	GATE 2021	A
26	GATE 2021	B
27	GATE 2022	A
28	GATE 2017	D
29	GATE 2023	A
30	GATE 2023	15
31	GATE 2024	2
32	GATE 2025	A
33	GATE 2025	A
34	GATE 2026	A

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