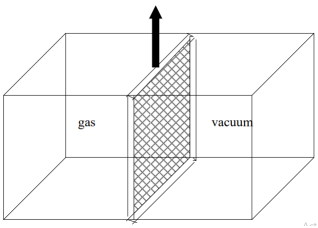
Classical Statistics TIFR

https://snboseplc.com/classical-statistics-tifr/

TIFR Questions

Q.No:1 TIFR-2012

Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure).



If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

(a) $2k_B$
(b) $k_B \ln(1/2)$
(c) $k_B \ln 2$
(d) $(k_B \ln 2)/2$

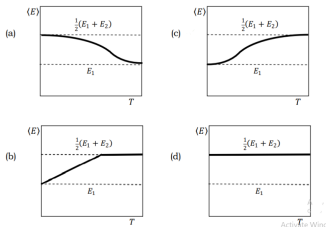
Q.No:2 TIFR-2012

When a gas is enclosed in an impermeable box and heated to a high temperature T, some of the neutral atoms lose an electron and become ions. If the number density of neutral atoms, ions and electrons is N_a, N_i and N_e, respectively, these can be related to the average volume V_a occupied by an atom/ion and the ionisation energy E by the relation

(a) $N_e(N_a + N_i) = (N_a/V_a)\exp(-E/k_BT)$
(b) $N_a(N_e + N_i) = (N_a/V_a)\exp(-E/k_BT)$
(c) $N_eN_i = (N_a/V_a)\exp(+E/k_BT)$
(d) $N_eN_i = (N_a/V_a)\exp(-E/k_BT)$

Q.No:3 TIFR-2013

Consider an ensemble of microscopic quantum mechanical systems with two energy levels E_1 and E_2, where $E_1 < E_2$. Which of the following graphs best describes the temperature dependence of the average energy $\langle E\rangle$ of the system?



Q.No:4 TIFR-2013

A system at temperature T has three energy states $0, \pm\varepsilon$. The entropy of the system in the low temperature ($T \to 0$) and high temperature ($T \to \infty$) limits are, respectively,

(a) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_B \exp(-3)$
(b) $S_{T\to 0} = S_{T\to\infty} = k_B \ln 3$
(c) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_B \ln 3$
(d) $S_{T\to 0} = 0$ and $S_{T\to\infty} = 3k_B/2$

Q.No:5 TIFR-2014

Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat C_V of CO gas is given in terms of the Boltzmann constant k_B by

(a) $\frac{5}{2}k_B$
(b) $2k_B$
(c) $\frac{3}{2}k_B$
(d) $\frac{1}{2}k_B$

Q.No:6 TIFR-2015

In a monatomic gas, the first excited state is only $1.5eV$ above the ground state, while the other excited states are much higher up. The ground state is doubly-degenerate, while the first excited state has a four-fold degeneracy. If now, the gas is heated to a temperature of $7000K$, the fraction of atoms in the excited state will be approximately

(a) 0.14
(b) 0.07
(c) 0.42
(d) 0.3

Q.No:7 TIFR-2016

N distinguishable particles are distributed among three states having energies $E = 0, k_BT$ and $2k_BT$ respectively. If the total equilibrium energy of the system is $138.06k_BT$, find the number N of particles.

Q.No:8 TIFR-2017

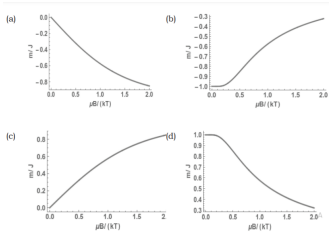
Consider a system of non-interacting particles with integer angular momentum J at a temperature T. This

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system is placed in a magnetic field B in the z direction. The energy of a state with $J_z = m\hbar$ is

$$E_m = m\mu_B B$$

with $\mu_B > 0$. The fractional magnetization of the particles as a function of $\mu_B B/k_B T$ can be represented as



Q.No:9 TIFR-2017

Two identical bosons may occupy any of two energy levels $0, \varepsilon$, where $\varepsilon > 0$. The lowest energy state is doubly-degenerate and the excited state is non-degenerate. Assume that the two-particle system is in thermal equilibrium at a temperature T. Calculate the average energy $\langle E \rangle$. What will be the leading term of

$$\frac{\langle E \rangle}{\exp\left(-\frac{\varepsilon}{k_B T}\right)}$$

at low temperature?

Q.No:10 TIFR-2017

A one-dimensional quantum harmonic oscillator of natural frequency ω is in thermal equilibrium with a heat bath at temperature T. The mean value $\langle E \rangle$ of the energy of the oscillator can be written as

(a) $\frac{\hbar\omega}{2}\text{sech}\left(\frac{\hbar\omega}{2k_B T}\right)$
(b) $\frac{\hbar\omega}{2}\text{csch}\left(\frac{\hbar\omega}{2k_B T}\right)$
(c) $\frac{\hbar\omega}{2}\coth\left(\frac{\hbar\omega}{2k_B T}\right)$
(d) $\frac{\hbar\omega}{2}\tanh\left(\frac{\hbar\omega}{2k_B T}\right)$

Q.No:11 TIFR-2017

Hydrogen atoms in the atmosphere of a star are in thermal equilibrium, with an average kinetic energy of 1 eV. The ratio of the number of hydrogen atoms in the 2nd excited state ($n = 3$) to the number in the ground state ($n = 1$) is

(a) 3.16×10^{-11}
(b) 1.33×10^{-8}
(c) 3.16×10^{-8}
(d) 5.62×10^{-6}

Q.No:12 TIFR-2018

A classical ideal gas of atoms with masses m is confined in a three-dimensional potential

$$V(x, y, z) = \frac{\lambda}{2}(x^2 + y^2 + z^2)$$

at a temperature T. If k_B is the Boltzmann constant, the root mean square (r.m.s.) distance of the atoms from the origin is

(a) $\left(\frac{3k_B T}{\lambda}\right)^{1/2}$
(b) $\left(\frac{3k_B T}{2\lambda}\right)^{1/2}$
(c) $\left(\frac{2k_B T}{3\lambda}\right)^{1/2}$
(d) $\left(\frac{k_B T}{\lambda}\right)^{1/2}$

Q.No:13 TIFR-2018

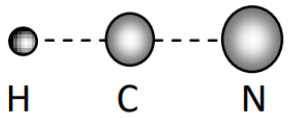
N particles are distributed among three energy levels having energies: $0, k_B T$ and $2k_B T$ respectively. If the total equilibrium energy of the system is approximately $42.5k_B T$ then find the value of N (to the closest integer).

Q.No:14 TIFR-2018

A statistical system, kept at a temperature T, has n discrete energy levels with equal level-spacing ε, starting from energy 0. If, now, a single particle is placed in the system what will be the mean energy of the system in the limit as $n \to \infty$? [The answer should not be left as a summation]

Q.No:15 TIFR-2019

Consider the following linear model of a molecule of hydrogen cyanide (HCN) depicted below.



It follows that the molar specific heat of hydrogen cyanide gas at constant pressure must be

(a) $6R$
(b) $4.5R$
(c) $5R$
(d) $5.5R$

Q.No:16 TIFR-2019

Consider N non-interacting distinguishable particles in equilibrium at an absolute temperature T. Each particle can only occupy one of two possible states of energy 0 and ϵ respectively ($\epsilon > 0$). The entropy of the system, in terms of $\beta = \epsilon/k_B T$ is

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(a) $Nk_B\left\{\ln(1+e^{-\beta})-\frac{e^{-\beta}}{1+e^{-\beta}}\right\}$
(b) $Nk_B\left\{\ln(1+e^{-\beta})+\frac{\beta e^{-\beta}}{1+e^{-\beta}}\right\}$
(c) $Nk_B\left\{\ln(1-e^{-\beta})+\frac{\beta e^{-\beta}}{1-e^{-\beta}}\right\}$
(d) $Nk_B\left\{\ln(1+e^{-\beta})-\frac{e^{-\beta}}{1-e^{-\beta}}\right\}$

Q.No:17 TIFR-2020

In a certain atom, the ground state and first excited state of the valence electron are -7.8 eV and -3.9 eV, while all the higher excited states have energies very close to zero. The ground state has a degeneracy of 2, while the first excited state has a degeneracy of 6.

It follows that if these atoms reside in the outer layers of a blue giant star at a temperature around 2.32×10^4 K, the average energy per atom will be approximately
(a) -5.1 eV
(b) -5.9 eV
(c) -6.8 eV
(d) -4.4 eV

Q.No:18 TIFR-2020

A square lattice consists of $2N$ sites, of which alternate sites are labeled A and B. An example with $N = 6$ is shown on the right. Now, N identical classical particles are distributed over these sites, such that each site can accommodate at most one particle.

A	B	A	B	A	B
B	A	B	A	B	A
A	B	A	B	A	B
B	A	B	A	B	A
A	B	A	B	A	B
B	A	B	A	B	A

The fraction of the total number N of particles occupying A sites is denoted α and the fraction occupying B sites is denoted β, so that $\alpha+\beta=1$.

If α, β are fixed and $N \gg 1$, the entropy S of the system can be written
(a) $S=-2Nk_BT(\alpha\ln\alpha+\beta\ln\beta)$
(b) $S=2Nk_BT(\alpha\ln\alpha+\beta\ln\beta)$
(c) $S=-2Nk_BT(\alpha\ln\alpha-\beta\ln\beta)$
(d) $S=2Nk_BT(\alpha\ln\alpha-\beta\ln\beta)$

Q.No:19 TIFR-2020

A system is composed of a large number of non-interacting classical particles moving in two dimensions, which individually obey the Hamiltonian

$$\frac{p_x^2+p_y^2}{2m}+\frac{1}{2}m\omega^2(x^2+y^2)$$

and the system is connected to a heat bath at a temperature T.

The probability of finding a particle within a radius R from the origin is given by
(a) $1-\exp\left(-\frac{m\omega^2R^2}{2T}\right)$
(b) $\exp\left(-\frac{m\omega^2R^2}{2T}\right)$
(c) $\mathrm{erf}\left(\sqrt{\frac{m}{2T}}\omega R\right)$
(d) $1-\frac{m\omega^2R^2}{2T}$

Q.No:20 TIFR-2021

Which of the following is the entropy generated when two identical blocks at temperatures $2T$ and T are brought into thermal contact and allowed to reach equilibrium?
[Assume that the heat capacity of each block is C]
(a) $C(2\ln3-3\ln2)$
(b) zero
(c) $2C\ln\frac{3}{2}$
(d) $C(2\ln2-3\ln3)$

Q.No:21 TIFR-2021

A certain system has one state with energy E, two states with energy $2E$, three states with energy $3E$ and so on, where $E>0$. The partition function Z of the system at temperature T is given by
(a) $\frac{1}{Z}=4\sinh^2\frac{E}{2T}$
(b) $\frac{1}{Z}=2\cosh^2\frac{E}{4T}$
(c) $\frac{1}{Z}=4\coth^2\frac{E}{2T}$
(d) $\frac{1}{Z}=2\tanh^2\frac{E}{4T}$

Q.No:22 TIFR-2022

A vertical cylinder of height H is filled with an ideal gas of classical point particles each of mass m and is allowed to come to equilibrium under gravity at a temperature T. The mean height of these particles is
(a) $\frac{k_BT}{mg}\ \left(1-\frac{mgH/k_BT}{e^{mgh/k_BT}-1}\right)$
(b) $\frac{H}{3}\ \frac{mgH/k_BT}{e^{mgh/k_BT}+1}$
(c) $\frac{k_BT}{mg}\ \left(1-\frac{mgH/k_BT}{e^{mgh/k_BT}+1}\right)$
(d) $\frac{H}{3}\ \frac{mgH/k_BT}{e^{mgh/k_BT}-1}$

Q.No:23 TIFR-2023

A two-level system with zero ground state energy is in equilibrium at a nonzero finite temperature. If α is defined as the ratio

$$\alpha=\frac{\langle E^2\rangle}{\langle E\rangle^2}$$

where $\langle E\rangle$ denotes mean energy and $\langle E^2\rangle$ denotes mean squared energy, then
(a) $2<\alpha<\infty$
(b) $0<\alpha<\frac{1}{2}$
(c) $\frac{1}{2}<\alpha<1$
(d) $1<\alpha\leq2$

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Q.No:24 TIFR-2024

A collection of N spin-$\frac{1}{2}$ objects, with individual energy eigenvalues $0, E$, are kept at a temperature T. Which of the following curves accurately represent the temperature dependence of specific heat of the system?

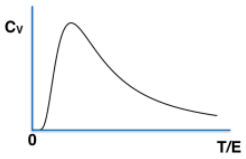


Figure 1: Option (a)

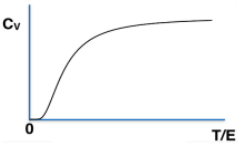


Figure 2: Option (b)

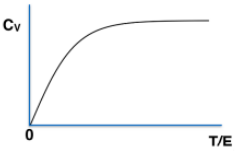


Figure 3: Option (c)

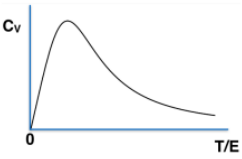
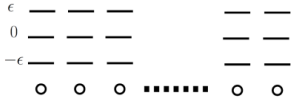


Figure 4: Option (d)

Q.No:25 TIFR-2024

Each site of a linear chain of N sites has a spin which can be in three different states with energies $0, \pm\epsilon$, as shown in the figure below.



The system has a constraint that the neighboring spins cannot be in the same state. At infinite temperature, the entropy of the system is given by:

(a) $N \ln 2 + \ln \frac{3}{2}$
(b) $N \ln 3$
(c) $(N-1) \ln 2$
(d) $N \ln 2$

Q.No:26 TIFR-2024

A string has 8 beads in a row, with n identical red beads and $(8-n)$ identical blue beads. When one of the red beads is replaced by a blue one, the entropy of the given system changes from S to $S + k_B \ln 2$. All configurations of the beads are equally probable. What is the value of n?

(a) 6
(b) 2
(c) 4
(d) 8

Q.No:27 TIFR-2025

Consider two ideal gases A and B with atomic masses m_A and m_B respectively such that $m_A > m_B$. The two gases with the same number of moles are kept at the same temperature and confined in containers with the same volume. Which of the gases will exert more pressure and molecules of which gas will have a higher RMS momentum?

a) Both will exert the same pressure but molecules of Gas A will have more RMS momentum
b) Gas A will exert more pressure and molecules of Gas B will have more RMS momentum
c) Gas B will exert more pressure but molecules of Gas A will have more RMS momentum
d) Both will exert the same pressure and molecules of both gases have the same RMS momentum

Q.No:28 TIFR-2025

A classical ideal gas at temperature T is placed in a spherically symmetric potential

$$V(r) = c\, r^3$$

What is $\langle V(r) \rangle$ per particle?
a) kT
b) $\frac{3kT}{2}$
c) $\frac{kT}{2}$
d) $\frac{kT}{3}$

Q.No:29 TIFR-2026

A non-interacting, classical gas is made up of N Nitrogen molecules. The specific heat at constant volume is C_v. $C_v/(Nk_BT)$ is given by:
(Ignore rotations about the axis of symmetry.)

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(a) 3/2 for some very low T, 5/2 for temperatures around the room temperature, 7/2 for some high temperatures

(b) 7/2 for some very low T, 5/2 for temperatures around the room temperature, 3/2 for some high temperatures

(c) 3/2 for some very low T, 2 for temperatures around the room temperature, 5/2 for some high temperatures

(d) 3/2 for some very low T, 2 for temperatures around the room temperature, 3 for some high temperatures

Q.No:30 TIFR-2026

Consider a single particle in a potential well with an energy spectrum

$$E_n = n\epsilon, \qquad n = 0, 1, 2, 3, \ldots,$$

with $\epsilon > 0$. The n^{th} energy level has a degeneracy

$$g(n) = 2n + 1.$$

This system is at temperature T. What is the value of $\langle \hat{H} \rangle$?

(k is the Boltzmann constant.)

(a) $\frac{\epsilon(3e^{\epsilon/kT}+1)}{(e^{2\epsilon/kT}-1)}$

(b) $\frac{\epsilon}{(e^{\epsilon/kT}-1)}$

(c) $\frac{\epsilon(3e^{\epsilon/kT}+1)}{(e^{\epsilon/kT}-1)^2}$

(d) $\frac{\epsilon}{(e^{\epsilon/kT}-1)^2}$

Q.No:31 TIFR-2026

A system of N classical non-interacting particles of mass m is confined to a cubic box of volume V. Inside the box is a region of volume $V_0 (< V)$, where a constant potential U is present.

If P is the pressure of the system, T is its temperature, then:

(a) $\frac{P}{Nk_BT} = \frac{1}{V - V_0(1 - e^{-U/k_BT})}$

(b) $\frac{P}{Nk_BT} = \frac{1}{V - V_0 e^{-U/k_BT}}$

(c) $\frac{P}{Nk_BT} = \frac{1}{V_0 e^{-U/k_BT}}$

(d) $\frac{P}{Nk_BT} = \frac{1}{V - V_0}$

Question No.	Entrance with year	
1	TIFR 2012	C
2	TIFR 2012	D
3	TIFR 2013	C
4	TIFR 2013	C
5	TIFR 2014	A
6	TIFR 2015	A
7	TIFR 2016	325
8	TIFR 2017	A
9	TIFR 2017	$2\varepsilon/3$
10	TIFR 2017	C
11	TIFR 2017	B
12	TIFR 2018	A
13	TIFR 2018	100
14	TIFR 2018	$\frac{\varepsilon}{e^{\varepsilon/k_BT} - 1}$
15	TIFR 2019	D
16	TIFR 2019	B
17	TIFR 2020	C
18	TIFR 2020	A
19	TIFR 2020	A
20	TIFR 2021	A
21	TIFR 2021	A
22	TIFR 2022	A
23	TIFR 2023	A
24	TIFR 2024	A
25	TIFR 2024	A
26	TIFR 2024	A
27	TIFR 2025	A
28	TIFR 2025	A
29	TIFR 2026	A
30	TIFR 2026	A
31	TIFR 2026	A

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