Classical Statistics CSIR

https://snboseplc.com/classical-statistics-csir/

CSIR Questions

Q.No:1 CSIR Dec-2014

In low density oxygen gas at low temperature, only the translational and rotational modes of the molecules are excited. The specific heat per molecule of the gas is

(1) $\frac{1}{2}k_B$

(2) k_B

(3) $\frac{3}{2}k_B$

(4) $\frac{5}{2}k_B$

Q.No:2 CSIR Dec-2014

A collection N of non-interacting spins S_i, $i = 1, 2, \cdots, N$, $(S_i = \pm 1)$ is kept in an external magnetic field B at a temperature T. The Hamiltonian of the system is $H = -\mu B \sum_i S_i$. What should be the minimum value of $\frac{\mu B}{k_B T}$ for which the mean value $\langle S_i \rangle \geq \frac{1}{3}$?

(1) $\frac{1}{2}N \ln 2$

(2) $2 \ln 2$

(3) $\frac{1}{2} \ln 2$

(4) $N \ln 2$

Q.No:3 CSIR June-2015

A system of N distinguishable particles, each of which can be in one of the two energy levels 0 and ϵ, has a total energy $n\epsilon$, where n is an integer. The entropy of the system is proportional to

(1) $N \ln n$

(2) $n \ln N$

(3) $\ln\left(\frac{N!}{n!}\right)$

(4) $\ln\left(\frac{N!}{n!(N-n)!}\right)$

Q.No:4 CSIR June-2015

A system of N non-interacting classical particles, each of mass m is in a two dimensional harmonic potential of the form $V(r) = \alpha(x^2 + y^2)$ where α is a positive constant. The canonical partition function of the system at temperature T is $\left(\beta = \frac{1}{k_B T}\right)$:

(1) $\left[\left(\frac{\alpha}{2m}\right)^2 \frac{\pi}{\beta}\right]^N$

(2) $\left(\frac{2m\alpha}{\alpha\beta}\right)^{2N}$

(3) $\left(\frac{\alpha\pi}{2m\beta}\right)^N$

(4) $\left(\frac{2m\pi^2}{\alpha\beta^2}\right)^N$

Q.No:5 CSIR June-2015

Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength J. The partition function of the system at temperature T is given by $\left(\beta = \frac{1}{k_B T}\right)$:

(1) $2e^{3\beta J} + 6e^{-\beta J}$

(2) $2e^{-3\beta J} + 6e^{\beta J}$

(3) $2e^{3\beta J} + 6e^{-3\beta J} + 3e^{\beta J} + 3e^{-\beta J}$

(4) $(2\cosh \beta J)^3$

Q.No:6 CSIR Dec-2015

For a system of independent non-interacting one-dimensional oscillators, the value of the free energy per oscillator, in the limit $T \to 0$, is

(1) $\frac{1}{2}\hbar\omega$

(2) $\hbar\omega$

(3) $\frac{3}{2}\hbar\omega$

(4) 0

Q.No:7 CSIR Dec-2015

The partition function of a system of N Ising spins is $Z = \lambda_1^N + \lambda_2^N$, where λ_1 and λ_2 are functions of temperature, but are independent of N. If $\lambda_1 > \lambda_2$, the free energy per spin in the limit $N \to \infty$ is

(1) $-k_B T \ln\left(\frac{\lambda_1}{\lambda_2}\right)$

(2) $-k_B T \ln \lambda_2$

(3) $-k_B T \ln(\lambda_1 \lambda_2)$

(4) $-k_B T \ln \lambda_1$

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Q.No:8 CSIR Dec-2015

The Hamiltonian of a system of N non-interacting spin-1/2 particles is $H = -\mu_0 B \sum_i S_i^Z$, where $S_i^Z = \pm 1$ are the components of i^{th} spin along an external magnetic field B. At a temperature T such that $e^{\mu_0 B/k_B T} = 2$, the specific heat per particle is

(1) $\frac{16}{25} k_B$

(2) $\frac{8}{25} k_B \ln 2$

(3) $k_B (\ln 2)^2$

(4) $\frac{16}{25} k_B (\ln 2)^2$

Q.No:9 CSIR Dec-2015

An ensemble of non-interacting spin-1/2 particles is in contact with a heat bath at temperature T, and is subjected to an external magnetic field. Each particle can be in one of the two quantum states of energies $\pm\epsilon_0$. If the mean energy per particle is $-\epsilon_0/2$, then the free energy per particle is

(1) $-2\epsilon_0 \frac{\ln(4/\sqrt{3})}{\ln 3}$

(2) $-\epsilon_0 \ln(3/2)$

(3) $-2\epsilon_0 \ln 2$

(4) $-\epsilon_0 \frac{\ln 2}{\ln 3}$

Q.No:10 CSIR June-2016

The specific heat per molecule of a gas of diatomic molecules at high temperatures is

(1) $8k_B$

(2) $3.5k_B$

(3) $4.5k_B$

(4) $3k_B$

Q.No:11 CSIR June-2016

A gas of non-relativistic classical particles in one dimension is subjected to a potential $V(x) = \alpha|x|$ (where α is a constant). The partition function is ($\beta = \frac{1}{k_B T}$)

(1) $\sqrt{\frac{4m\pi}{\beta^3 \alpha^2 h^2}}$

(2) $\sqrt{\frac{2m\pi}{\beta^3 \alpha^2 h^2}}$

(3) $\sqrt{\frac{8m\pi}{\beta^3 \alpha^2 h^2}}$

(4) $\sqrt{\frac{3m\pi}{\beta^3 \alpha^2 h^2}}$

Q.No:12 CSIR June-2016

Consider a gas of Cs atoms at a number density of 10^{12} atoms/cc. When the typical inter-particle distance is equal to the thermal de Broglie wavelength of the particles, the temperature of the gas is nearest to (Take the mass of a Cs atom to be 22.7×10^{-26} kg.)

(1) 1×10^{-9} K

(2) 7×10^{-5} K

(3) 1×10^{-3} K

(4) 2×10^{-8} K

Q.No:13 CSIR June-2016

The internal energy $E(T)$ of a system at a fixed volume is found to depend on the temperature T as $E(T) = aT^2 + bT^4$. Then the entropy $S(T)$, as a function of temperature, is

(1) $\frac{1}{2}aT^2 + \frac{1}{4}bT^4$

(2) $2aT^2 + 4bT^4$

(3) $2aT + \frac{4}{3}bT^3$

(4) $2aT + 2bT^3$

Q.No:14 CSIR Dec-2016

Consider a gas of N classical particles in a two-dimensional square box of side L. If the total energy of the gas is E, the entropy (apart from an additive constant) is

(1) $Nk_B \ln\left(\frac{L^2 E}{N}\right)$

(2) $Nk_B \ln\left(\frac{LE}{N}\right)$

(3) $2Nk_B \ln\left(\frac{L^2\sqrt{E}}{N}\right)$

(4) $L^2 k_B \ln\left(\frac{E}{N}\right)$

Q.No:15 CSIR Dec-2016

The partition function of a two-level system governed by the Hamiltonian $H = \begin{bmatrix} \gamma & -\delta \\ -\delta & -\gamma \end{bmatrix}$ is

(1) $2\sinh(\beta\sqrt{\gamma^2 + \delta^2})$

(2) $2\cosh(\beta\sqrt{\gamma^2 + \delta^2})$

(3) $\frac{1}{2}\left[\cosh(\beta\sqrt{\gamma^2 + \delta^2}) + \sinh(\beta\sqrt{\gamma^2 + \delta^2})\right]$

(4) $\frac{1}{2}\left[\cosh(\beta\sqrt{\gamma^2 + \delta^2}) - \sinh(\beta\sqrt{\gamma^2 + \delta^2})\right]$

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Q.No:16 CSIR Dec-2016

An atom has a non-degenerate ground-state and a doubly-degenerate excited state. The energy difference between the two states is ε. The specific heat at very low temperatures ($\beta\varepsilon \gg 1$) is given by

(1) $k_B(\beta\varepsilon)$

(2) $k_B e^{-\beta\varepsilon}$

(3) $2k_B(\beta\varepsilon)^2 e^{-\beta\varepsilon}$

(4) k_B

Q.No:17 CSIR Dec-2016

The total spin of a hydrogen atom is due to the contribution of the spins of the electron and the proton. In the high temperature limit, the ratio of the number of atoms in the spin-1 state to the number in the spin-0 state is

(1) 2

(2) 3

(3) 1/2

(4) 1/3

Q.No:18 CSIR June-2017

In a thermodynamic system in equilibrium, each molecule can exist in three possible states with probabilities $1/2, 1/3$ and $1/6$ respectively. The entropy per molecule is

(1) $k_B \ln 3$

(2) $\frac{1}{2}k_B \ln 2 + \frac{2}{3}k_B \ln 3$

(3) $\frac{2}{3}k_B \ln 2 + \frac{1}{2}k_B \ln 3$

(4) $\frac{1}{2}k_B \ln 2 + \frac{1}{6}k_B \ln 3$

Q.No:19 CSIR June-2017

The Hamiltonian for three Ising spins S_0, S_1 and S_2, taking values ± 1, is

$$H = -JS_0(S_1 + S_2).$$

If the system is in equilibrium at temperature T, the average energy of the system, in terms of $\beta = (k_B T)^{-1}$, is

(1) $-\frac{1+\cosh(2\beta J)}{2\beta \sinh(2\beta J)}$

(2) $-2J[1 + \cosh(2\beta J)]$

(3) $-2/\beta$

(4) $-2J\frac{\sinh(2\beta J)}{1+\cosh(2\beta J)}$

Q.No:20 CSIR Dec-2017

A closed system having three non-degenerate energy levels with energies $E = 0, \pm\epsilon$, is at temperature T. For $\epsilon = 2k_B T$, the probability of finding the system in the state with energy $E = 0$, is

(1) $1/(1 + 2\cosh 2)$

(2) $1/(2\cosh 2)$

(3) $\frac{1}{2}\cosh 2$

(4) $1/\cosh 2$

Q.No:21 CSIR Dec-2017

Two non-degenerate energy levels with energies 0 and ϵ are occupied by N non-interacting particles at a temperature T. Using classical statistics, the average internal energy of the system is

(1) $N\epsilon/(1 + e^{\epsilon/k_B T})$

(2) $N\epsilon/(1 - e^{\epsilon/k_B T})$

(3) $N\epsilon e^{-\epsilon/k_B T}$

(4) $\frac{3}{2}Nk_B T$

Q.No:22 CSIR June-2018

In a system of N distinguishable particles, each particle can be in one of two states with energies 0 and $-E$, respectively. The mean energy of the system at temperature T is

(1) $-\frac{1}{2}N(1 + e^{E/k_B T})$

(2) $-NE/(1 + e^{E/k_B T})$

(3) $-\frac{1}{2}NE$

(4) $-NE/(1 + e^{-E/k_B T})$

Q.No:23 CSIR Dec-2018

The rotational energy levels of a molecule are $E_\ell = \frac{\hbar^2}{2I_0}\ell(\ell+1)$, where $\ell = 0, 1, 2, \cdots$ and I_0 is its moment of inertia. The contribution of the rotational motion to the Helmholtz free energy per molecule, at low temperatures in a dilute gas of these molecules, is approximately

(1) $-k_B T\left(1 + \frac{\hbar^2}{I_0 k_B T}\right)$

(2) $-k_B T e^{-\frac{\hbar^2}{I_0 k_B T}}$

(3) $-k_B T$

(4) $-3k_B T e^{-\frac{\hbar^2}{I_0 k_B T}}$

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Q.No:24 CSIR Dec-2018

The vibrational motion of a diatomic molecule may be considered to be that of a simple harmonic oscillator with angular frequency ω. If a gas of these molecules is at a temperature T, what is the probability that a randomly picked molecule will be found in its lowest vibrational state?

(1) $1 - e^{-\frac{\hbar\omega}{k_B T}}$

(2) $e^{-\frac{\hbar\omega}{2k_B T}}$

(3) $\tanh\left(\frac{\hbar\omega}{k_B T}\right)$

(4) $\frac{1}{2}\text{cosech}\left(\frac{\hbar\omega}{2k_B T}\right)$

Q.No:25 CSIR Dec-2018

The Hamiltonian of a one-dimensional Ising model of N spins (N large) is

$$H = -J\sum_{i=1}^{N}\sigma_i\sigma_{i+1}$$

where the spin $\sigma_i = \pm 1$ and J is a positive constant. At inverse temperature $\beta = 1/k_B T$, the correlation function between the nearest neighbour spins $\langle\sigma_i\sigma_{i+1}\rangle$ is

(1) $e^{-\beta J}/(e^{\beta J} + e^{-\beta J})$

(2) $e^{-2\beta J}$

(3) $\tanh(\beta J)$

(4) $\coth(\beta J)$

Q.No:26 CSIR Dec-2018

The energy levels accessible to a molecule have energies $E_1 = 0, E_2 = \Delta$ and $E_3 = 2\Delta$ (where Δ is a constant). A gas of these molecules is in thermal equilibrium at temperature T. The specific heat at constant volume in the high temperature limit ($k_B T \gg \Delta$) varies with temperature as

(1) $1/T^{3/2}$

(2) $1/T^3$

(3) $1/T$

(4) $1/T^2$

Q.No:27 CSIR June-2019

In a system comprising of approximately 10^{23} distinguishable particles, each particle may occupy any of 20 distinct states. The maximum value of the entropy per particle is nearest to

(1) $20k_B$

(2) $3k_B$

(3) $10(\ln 2)k_B$

(4) $20(\ln 2)k_B$

Q.No:28 CSIR June-2019

Consider a classical gas in thermal equilibrium at temperatures T_1 and T_2, where $T_1 < T_2$. Which of the following graphs correctly represents the qualitative behaviour of the probability density function of the x-component of the velocity?

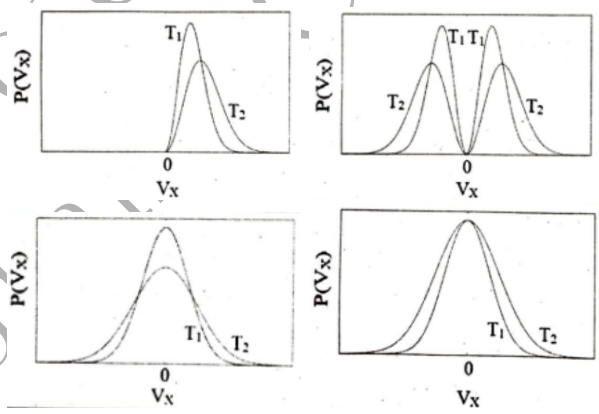


Figure 1: Graphs for Q.No:28

Q.No:29 CSIR June-2019

The Hamiltonian of a classical nonlinear one dimensional oscillator is $H = \frac{1}{2m}p^2 + \lambda x^4$, where $\lambda > 0$ is a constant. The specific heat of a collection of N independent such oscillators is

(1) $3Nk_B/2$

(2) $3Nk_B/4$

(3) Nk_B

(4) $Nk_B/2$

Q.No:30 CSIR June-2019

The Hamiltonian of three Ising spins S_1, S_2 and S_3, each taking values ± 1, is $H = -J(S_1S_2 + S_2S_3) - hS_1$, where J and h are positive constants. The mean value of S_3 in equilibrium at a temperature $T = 1/(k_B\beta)$, is

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(1) $\tanh^3(\beta J)$

(2) $\tanh(\beta h)\tanh^2(\beta J)$

(3) $\sinh(\beta h)\sinh^2(\beta J)$

(4) 0

Q.No:31 CSIR Dec-2019

The energies available to a three-state system are $0, E$ and $2E$, where $E > 0$. Which of the following graphs best represents the temperature dependence of the specific heat?

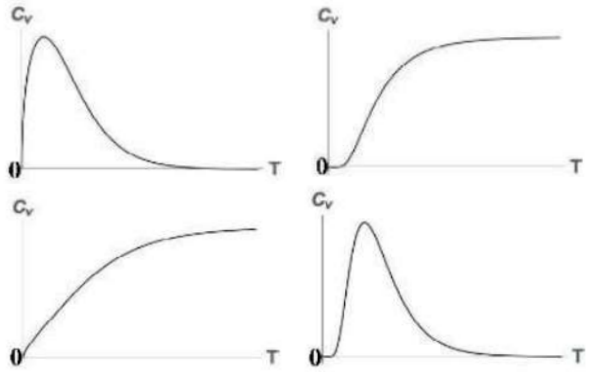


Figure 2: Graphs for Q.No:31

Q.No:32 CSIR Dec-2019

The angular frequency of oscillation of a quantum harmonic oscillator in two dimensions is ω. If it is in contact with an external heat bath at temperature T, its partition function is (in the following $\beta = \frac{1}{k_B T}$)

(1) $\frac{e^{2\beta\hbar\omega}}{(e^{2\beta\hbar\omega}-1)^2}$

(2) $\frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2}$

(3) $\frac{e^{\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$

(4) $\frac{e^{2\beta\hbar\omega}}{e^{2\beta\hbar\omega}-1}$

Q.No:33 CSIR Dec-2019

The Hamiltonian of two particles, each of mass m, is

$$H(q_1, p_1; q_2, p_2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + k\left(q_1^2 + q_2^2 + \frac{1}{4}q_1 q_2\right),$$

where $k > 0$ is a constant. The value of the partition function

$$Z(\beta) = \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dp_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} dp_2 e^{-\beta H(q_1,p_1;q_2,p_2)}$$

is

(1) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{16}{15}}$

(2) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{15}{16}}$

(3) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{63}{64}}$

(4) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{64}{63}}$

Q.No:34 Assam CSIR Dec-2019

The total internal energy of a system of N distinguishable particles (where N is large) is U. Each particle may be in the ground state of energy 0, or in an excited state of energy ϵ. The entropy of the system is

(1) $k_B\left[N\ln N + \left(N - \frac{U}{\epsilon}\right)\ln\left(N - \frac{U}{\epsilon}\right) + \frac{U}{\epsilon}\ln\frac{U}{\epsilon}\right]$

(2) $k_B\left[N\ln N - \left(N - \frac{U}{\epsilon}\right)\ln\left(N - \frac{U}{\epsilon}\right) + \frac{U}{\epsilon}\ln\frac{U}{\epsilon}\right]$

(3) $k_B\left[N\ln N - \left(N - \frac{U}{\epsilon}\right)\ln\left(N - \frac{U}{\epsilon}\right) - \frac{U}{\epsilon}\ln\frac{U}{\epsilon}\right]$

(4) $k_B N\ln N$

Q.No:35 Assam CSIR Dec-2019

The Hamiltonian of a system of three Ising spins is $H = -J(S_1S_2 + S_2S_3)$, where each spin $S_i = \pm 1$, and $J > 0$. Then at thermal equilibrium at temperature T, the correlation function $\langle S_1 S_2\rangle$ is

(1) 0

(2) $\operatorname{sech}\frac{J}{k_B T}$

(3) $\tanh\frac{J}{k_B T}$

(4) $\left(\tanh\frac{J}{k_B T}\right)^2$

Q.No:36 CSIR June-2020

An idealised atom has a non-degenerate ground state at zero energy and a g-fold degenerate excited state of energy E. In a non-interacting system of N such atoms, the population of the excited state may exceed that of the ground state above a temperature $T > \frac{E}{2k_B\ln 2}$. The minimum value of g for which this is possible is

(a) 8

(b) 4

(c) 2

(d) 1

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Q.No:37 CSIR June-2020

The Hamiltonian of a system of N non-interacting particles, each of mass m, in one dimension is

$$H = \sum_{i=1}^{N} \left(\frac{p_i^2}{2m} + \frac{\lambda}{4} x_i^4 \right)$$

where $\lambda > 0$ is a constant and p_i and x_i are the momentum and position respectively of the i-th particle. The average internal energy of the system is

(a) $\frac{4}{3} k_B T$

(b) $\frac{3}{4} k_B T$

(c) $\frac{3}{2} k_B T$

(d) $\frac{1}{3} k_B T$

Q.No:38 CSIR June-2020

For an ideal gas consisting of N distinguishable particles in a volume V, the probability of finding exactly 2 particles in a volume $\delta V \ll V$, in the limit $N, V \to \infty$, is

(a) $2N\delta V/V$

(b) $(N\delta V/V)^2$

(c) $\frac{(N\delta V)^2}{2V^2} e^{-N\delta V/V}$

(d) $\left(\frac{\delta V}{V}\right)^2 e^{-N\delta V/V}$

Q.No:39 CSIR June-2020

The Hamiltonian of a system of 3 spins is $H = J(S_1 S_2 + S_2 S_3)$, where $S_i = \pm 1$ for $i = 1, 2, 3$. Its canonical partition function, at temperature T, is

(a) $2\left(2\sinh \frac{J}{k_B T}\right)^2$

(b) $2\left(2\cosh \frac{J}{k_B T}\right)^2$

(c) $2\left(2\cosh \frac{J}{k_B T}\right)$

(d) $2\left(2\cosh \frac{J}{k_B T}\right)^3$

Q.No:40 CSIR Feb-2022

The energy levels of a non-degenerate quantum system are $\epsilon_n = nE_0$, where E_0 is a constant and $n = 1, 2, 3, \ldots$. At a temperature T, the free energy F can be expressed in terms of the average energy E by

(1) $E_0 + k_B T \ln \frac{E}{E_0}$

(2) $E_0 + 2k_B T \ln \frac{E}{E_0}$

(3) $E_0 - k_B T \ln \frac{E}{E_0}$

(4) $E_0 - 2k_B T \ln \frac{E}{E_0}$

Q.No:41 CSIR Feb-2022

A polymer, made up of N monomers, is in thermal equilibrium at temperature T. Each monomer could be of length a or $2a$. The first contributes zero energy, while the second one contributes ϵ. The average length (in units of Na) of the polymer at temperature $T = \epsilon/k_B$ is

(1) $\frac{5+e}{4+e}$

(2) $\frac{4+e}{3+e}$

(3) $\frac{3+e}{2+e}$

(4) $\frac{2+e}{1+e}$

Q.No:42 CSIR Feb-2022

Balls of ten different colours labeled by $a = 1, 2, \ldots, 10$ are to be distributed among different coloured boxes. A ball can only go in a box of the same colour, and each box can contain at most one ball. Let n_a and N_a denote respectively, the numbers of balls and boxes of colour a. Assuming that $N_a \gg n_a \gg 1$, the total entropy (in units of the Boltzmann constant) can be best approximated by

(1) $\sum_a (N_a \ln N_a + n_a \ln n_a - (N_a - n_a) \ln(N_a - n_a))$

(2) $\sum_a (N_a \ln N_a - n_a \ln n_a - (N_a - n_a) \ln(N_a - n_a))$

(3) $\sum_a (N_a \ln N_a - n_a \ln n_a + (N_a - n_a) \ln(N_a - n_a))$

(4) $\sum_a (N_a \ln N_a + n_a \ln n_a + (N_a - n_a) \ln(N_a - n_a))$

Q.No:43 CSIR Sep-2022

An elastic rod has a low energy state of length L_{max} and high energy state of length L_{min}. The best schematic representation of the temperature (T) dependence of the mean equilibrium length $L(T)$ of the rod, is

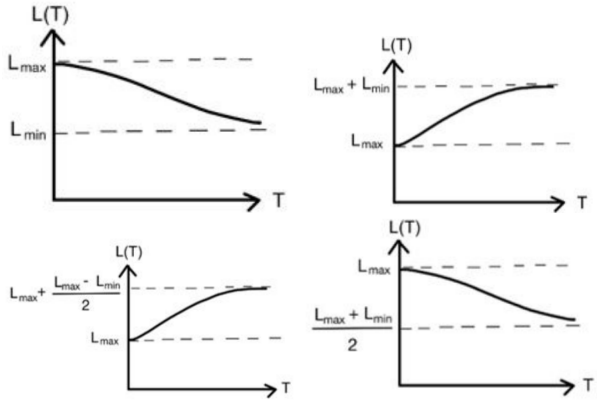


Figure 3: Graphs for Q.No:43

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Q.No:44 CSIR Sep-2022

If the average energy $\langle E\rangle_T$ of a quantum harmonic oscillator at a temperature T is such that $\langle E\rangle_T = 2\langle E\rangle_{T\to 0}$, then T satisfies

(1) $\coth\left(\frac{\hbar\omega}{k_BT}\right) = 2$

(2) $\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 2$

(3) $\coth\left(\frac{\hbar\omega}{k_BT}\right) = 4$

(4) $\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 4$

Q.No:45 CSIR Sep-2022

A paramagnetic salt with magnetic moment per ion $\mu_\pm = \pm\mu_B$ (where μ_B is the Bohr magneton) is in thermal equilibrium at temperature T in a constant magnetic field B. The average magnetic moment $\langle M\rangle$, as a function of k_BT/μ_BB is best represented by

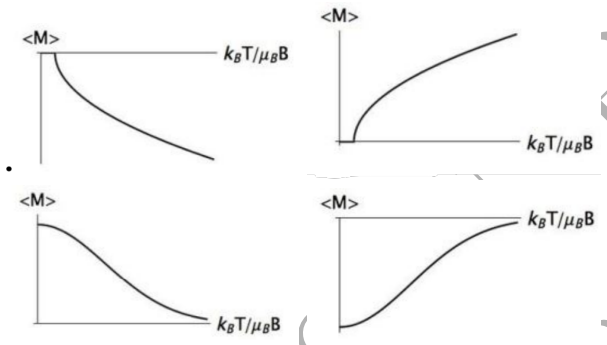


Figure 4: Graphs for Q.No:45

Q.No:46 CSIR Sep-2022

The energies of two level system are $\pm E$. Consider an ensemble of such non interacting systems at temperature T. At low temperatures, the leading term in the specific heat depend on T as

(1) $\frac{1}{T^2}e^{-E/k_BT}$

(2) $\frac{1}{T^2}e^{-2E/k_BT}$

(3) T^2e^{-E/k_BT}

(4) T^2e^{-2E/k_BT}

Q.No:47 CSIR Sep-2022

A system of N non-interacting particles in one-dimension, each of which is in a potential $V(x) = gx^6$ where $g > 0$ is a constant and x denotes the displacement of the particle from its equilibrium position. In thermal equilibrium, the heat capacity at constant volume is

(1) $\frac{7}{6}Nk_B$

(2) $\frac{4}{3}Nk_B$

(3) $\frac{3}{2}Nk_B$

(4) $\frac{2}{3}Nk_B$

Q.No:48 CSIR-Dec-2023

A quantum system is described by the Hamiltonian

$$H = JS_z + \lambda S_x,$$

where $S_i = \frac{\hbar}{2}\sigma_i$ and $\sigma_i (i = x, y, z)$ are the Pauli matrices. If $0 < \lambda \ll J$, then the leading correction in λ to the partition function of the system at temperature T is

(1) $\frac{\hbar\lambda^2}{2Jk_BT}\coth\left(\frac{J\hbar}{2k_BT}\right)$

(2) $\frac{\hbar\lambda^2}{2Jk_BT}\tanh\left(\frac{J\hbar}{2k_BT}\right)$

(3) $\frac{\hbar\lambda^2}{2Jk_BT}\cosh\left(\frac{J\hbar}{2k_BT}\right)$

(4) $\frac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\frac{J\hbar}{2k_BT}\right)$

Q.No:49 CSIR-Dec-2023

A system of N non-interacting classical spins, where each spin can take values $\sigma = -1, 0, 1$, is placed in a magnetic field h. The single spin Hamiltonian is given by

$$H = -\mu_Bh\sigma + \Delta(1 - \sigma^2),$$

where μ_B, Δ are positive constants with appropriate dimensions.

If M is the magnetization, the zero-field magnetic susceptibility per spin $\frac{1}{N}\frac{\partial M}{\partial h}\big|_{h\to 0}$, at a temperature $T = \frac{1}{\beta k_B}$ is given by

(1) $\beta\mu_B^2$

(2) $\frac{2\beta\mu_B^2}{2+e^{-\beta\Delta}}$

(3) $\beta\mu_B^2e^{-\beta\Delta}$

(4) $\frac{\beta\mu_B^2}{1+e^{-\beta\Delta}}$

Q.No:50 CSIR-June-2023

Two energy levels 0 (non-degenerate) and ϵ (doubly degenerate), are available to N non-interacting distinguishable particles. If U is the total energy of the system, for large values of N the entropy of the system is $k_B\left[N\ln N - \left(N - \frac{U}{\epsilon}\right)\ln\left(N - \frac{U}{\epsilon}\right) + X\right]$. In this expression, X is

(1) $-\frac{U}{\epsilon}\ln\frac{U}{2\epsilon}$

(2) $-\frac{U}{\epsilon}\ln\frac{2U}{\epsilon}$

(3) $-\frac{2U}{\epsilon}\ln\frac{2U}{\epsilon}$

(4) $-\frac{U}{\epsilon}\ln\frac{U}{\epsilon}$

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Q.No:51 CSIR-June-2023

In a one-dimensional system of N spins, the allowed values of each spin are $\sigma_i = \{1, 2, 3, \ldots, q\}$, where $q \geq 2$ is an integer. The energy of the system is

$$-J \sum_{i=1}^{N} \delta_{\sigma_i, \sigma_{i+1}}$$

where $J > 0$ is a constant. If periodic boundary conditions are imposed, the number of ground states of the system is

(1) q

(2) Nq

(3) q^N

(4) 1

Q.No:52 CSIR-June-2024

A single particle can exist in two states with energies 0 and E respectively. At high temperatures ($k_B T \gg E$), the specific heat of the system C_V will be approximately:

(1) proportional to $\frac{1}{T}$

(2) proportional to $\frac{1}{T^2}$

(3) proportional to $\frac{E}{e^{k_B T}}$

(4) constant

Q.No:53 CSIR-June-2024

Quantum particles of unit mass, in a potential

$$V(x) = \begin{cases} \frac{1}{2}\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \leq 0 \end{cases}$$

are in equilibrium at a temperature T. Let n_2 and n_3 denote the numbers of the particles in the second and third excited states respectively. The ratio n_2/n_3 is given by:

(1) $\exp\left(\frac{2\hbar\omega}{k_B T}\right)$

(2) $\exp\left(\frac{\hbar\omega}{k_B T}\right)$

(3) $\exp\left(\frac{3\hbar\omega}{k_B T}\right)$

(4) $\exp\left(\frac{4\hbar\omega}{k_B T}\right)$

Q.No:54 CSIR-June-2024

Rotational energy of a molecule in the angular momentum state j is given by

$$E_j = \frac{\hbar^2}{2I} j(j+1),$$

where I is the moment of inertia of the molecule. The probability that the molecule will be in its ground state at temperature T (such that $k_B T \gg \frac{\hbar^2}{2I}$) is:

(1) $\frac{3\hbar^2}{2Ik_B T}$

(2) $\frac{2\hbar^2}{3Ik_B T}$

(3) $\frac{\hbar^2}{2Ik_B T}$

(4) $\frac{\hbar^2}{Ik_B T}$

Q.No:55 CSIR-June-2024

Five classical spins are placed at the vertices of a regular pentagon. The Hamiltonian of the system is $H = J \sum S_i S_j$, where $J > 0$, $S_i = \pm 1$ and the sum is over all possible nearest neighbour pairs. The degeneracy of the ground state is

(1) 8

(2) 5

(3) 4

(4) 10

Q.No:56 CSIR-Dec-2024

An isolated box of volume V contains 5 identical, but distinguishable and non-interacting particles. The particles can either be in the ground state of zero energy or in an excited state of energy ε. The ground state is non-degenerate while the excited state is doubly degenerate. There is no restriction on the number of particles that can be put in a given state. The number of accessible microstates corresponding to the macrostate of the system with total energy $E = 2\varepsilon$ are

(1) 10

(2) 20

(3) 40

(4) 30

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Q.No:57 CSIR-Dec-2024

A system comprises N distinguishable atoms ($N \gg 1$). Each atom has two energy levels ω and 3ω ($\omega > 0$). Let ε_{eq} denote the average energy per particle when the system is in thermal equilibrium. The upper limit of ε_{eq} is

(1) $\frac{3\omega}{2}$

(2) 3ω

(3) $\frac{5\omega}{2}$

(4) 2ω

Q.No:58 CSIR-Dec-2024

Energy of two Ising spins ($s = \pm\frac{1}{2}$) is given by $E = s_1 s_2 + s_1 + s_2$.

At temperature T, the probability that both spins take the value $-\frac{1}{2}$ is 16 times the probability that both take the value $+\frac{1}{2}$. At the same temperature, what is the probability that the spins take opposite values?

(1) $\frac{16}{25}$

(2) $\frac{8}{25}$

(3) $\frac{8}{33}$

(4) $\frac{16}{33}$

Q.No:59 CSIR-Dec-2024

Eigenstates of a system are specified by two non-negative integers n_1 and n_2. The energy of the system is given by

$$E_n = \left(n_1 + \frac{1}{2}\right)\hbar\omega + \left(n_2 + \frac{1}{2}\right)2\hbar\omega.$$

If $\alpha \equiv \exp\left(-\frac{\hbar\omega}{k_B T}\right)$, what is the probability that at temperature T the energy of the system will be less than $4\hbar\omega$?

(1) $(1-\alpha^2)(1-\alpha)(2+\alpha+2\alpha^2)$

(2) $(1-\alpha)^2(1-\alpha)(2+\alpha+\alpha^2)$

(3) $(1-\alpha^2)(1+\alpha)(1+\alpha+2\alpha^2)$

(4) $(1-\alpha)^2(1+\alpha)(1+\alpha+2\alpha^2)$

Q.No:60 CSIR-June-2025

Consider one mole of an ideal diatomic gas molecule at temperature T such that $k_B T \gg h\nu$, where ν is the frequency of its vibrational mode. If C_p and C_v are the specific heats of this gas at constant pressure and volume respectively, then the ratio $\gamma = \frac{C_p}{C_v}$ is

(1) 2

(2) $\frac{7}{5}$

(3) $\frac{5}{3}$

(4) $\frac{9}{7}$

Q.No:61 CSIR-June-2025

There are two boxes, one at the ground level, and the other at a fixed height h. There are three balls of different colours, each having mass m and radius $r \ll h$. There is no restriction on the number of balls that can be simultaneously put in a given box.

For a given value of the total energy E (in units of mgh, g being the acceleration due to gravity), the number of accessible microstates is $\Omega(E)$. The plot of $\Omega(E)$ vs E is:

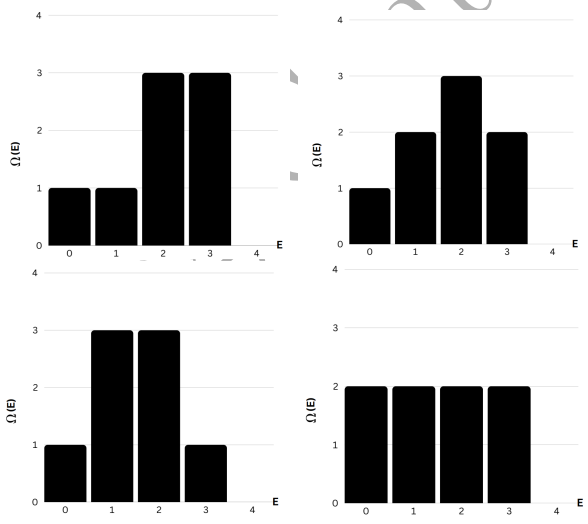


Figure 5: Graphs for Q.No:59

Q.No:62 CSIR-June-2025

Consider $2N$ Ising spins, s_i ($s_i = \pm 1$), in a one-dimensional lattice with periodic boundary conditions. The Hamiltonian is given by

$$H = -J\sum_{i=1}^{2N} s_i s_{i+1}$$

where J denotes the strength of the nearest-neighbour interactions with $J > 0$.

Let F be the fully ferromagnetic state and let A be the lowest-energy state with zero magnetization. The energy difference between these two states is

(1) $\frac{3J}{2}$

(2) $4J$

(3) $\frac{J}{2}$

(4) $2J$

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Q.No:63 CSIR-June-2025

A rigid molecule can have two possible rotational states: $j = 0$ or $j = 1$. Its rotational energies are given by

$$\epsilon_j = \frac{\hbar^2}{2I} j(j+1),$$

where I is its moment of inertia.

For an ensemble of such molecules in thermal equilibrium at temperature T, the ratio of the number of molecules in the $j = 1$ state (N_1) to those in the $j = 0$ state (N_0) is

$$\frac{N_1}{N_0} = 0.003.$$

The temperature T (in units of $\frac{\hbar^2}{2Ik_B}$) is closest to

(1) 0.29

(2) 0.21

(3) 0.15

(4) 0.34

Q.No:64 CSIR-Dec-2025

A classical mono-atomic ideal gas is in thermal equilibrium at temperature T. The velocity of a molecule of this gas, of mass m, is

$$\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}.$$

The value of the ensemble average $\langle v_x^2 v_y^2 \rangle$ is

1. $\left(\frac{k_BT}{2m}\right)^2$
2. $\left(\frac{k_BT}{m}\right)^2$
3. $\left(\frac{3k_BT}{2m}\right)^2$
4. $\left(\frac{2k_BT}{m}\right)^2$

Q.No:65 CSIR-Dec-2025

Consider a one-dimensional lattice (with lattice spacing a) along X-axis with sites labelled by $x = 0, 1, 2, \ldots, L$. A particle carrying a charge $-q$ can occupy any one of these sites. An electric field of strength E is applied in the positive x-direction. The average energy of the particle at a temperature T (in the limit $L \to \infty$) is ($\beta = \frac{1}{k_BT}$)

1. $\frac{Eqa}{e^{\beta Eqa}-1}$
2. $\frac{Eqa}{1+e^{\beta Eqa}}$
3. $\frac{Eqa}{2}$
4. $-Eqa$

Q.No:66 CSIR-June-2026

A micro-canonical ensemble consists of six non-interacting and distinguishable spin 1 particles in a uniform magnetic field. Each particle can thus be in energy states $-E_0$, 0 and E_0. The number of microstates in the state with total energy zero is:

1. 51
2. 191
3. 141
4. 55

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Question No.	**Entrance with year**	answer
1	CSIR Dec-2014	Option 4
2	CSIR Dec-2014	Option 3
3	CSIR June-2015	Option 4
4	CSIR June-2015	Option 4
5	CSIR June-2015	Option 1
6	CSIR Dec-2015	Option 1
7	CSIR Dec-2015	Option 4
8	CSIR Dec-2015	Option 4
9	CSIR Dec-2015	Option 1
10	CSIR June-2016	Option 2
11	CSIR June-2016	Option 3
12	CSIR June-2016	Option 3
13	CSIR June-2016	Option 3
14	CSIR Dec-2016	Option 3
15	CSIR Dec-2016	Option 2
16	CSIR Dec-2016	Option 3
17	CSIR Dec-2016	Option 2
18	CSIR June-2017	Option 3
19	CSIR June-2017	Option 4
20	CSIR Dec-2017	Option 1
21	CSIR Dec-2017	Option 1
22	CSIR June-2018	Option 4
23	CSIR Dec-2018	Option 4
24	CSIR Dec-2018	Option 1
25	CSIR Dec-2018	Option 3
26	CSIR Dec-2018	Option 4
27	CSIR June-2019	Option 2
28	CSIR June-2019	Option 3
29	CSIR June-2019	Option 2
30	CSIR June-2019	Option 2

Question No.	**Entrance with year**	answer
31	CSIR Dec-2019	Option 4
32	CSIR Dec-2019	Option 2
33	CSIR Dec-2019	Option 4
34	Assam CSIR Dec-2019	Option 3
35	Assam CSIR Dec-2019	Option 3
36	CSIR June-2020	Option b
37	CSIR June-2020	Option b
38	CSIR June-2020	Option c
39	CSIR June-2020	Option b
40	CSIR Feb-2022	Option 3
41	CSIR Feb-2022	Option 4
42	CSIR Feb-2022	Option 2
43	CSIR Sep-2022	Option 4
44	CSIR Sep-2022	Option 2
45	CSIR Sep-2022	Option 3
46	CSIR Sep-2022	Option 2
47	CSIR Sep-2022	Option 4
48	CSIR Dec-2023	Option 4
49	CSIR Dec-2023	Option 2
50	CSIR June-2023	Option 1
51	CSIR June-2023	Option 1
52	CSIR June-2024	Option 2
53	CSIR June-2024	Option 1
54	CSIR June-2024	Option 2
55	CSIR June-2024	Option 4
56	CSIR Dec-2024	Option 3
57	CSIR Dec-2024	Option 4
58	CSIR Dec-2024	Option 4
59	CSIR Dec-2024	Option 4
60	CSIR June-2025	Option 4
61	CSIR June-2025	Option 3
62	CSIR June-2025	Option 2
63	CSIR June-2025	Option 1
64	CSIR Dec-2025	Option 2
65	CSIR Dec-2025	Option 1
66	CSIR June-2026	Option 3

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