Quantum Statistics GATE

https://snboseplc.com/quantum-statistics-gate/

GATE Questions

Q.No:1 GATE-2012

For an ideal Fermi gas in three dimensions, the electron velocity v_F at the Fermi surface is related to electron concentration n as,

(A) $v_F \propto n^{2/3}$
(B) $v_F \propto n$
(C) $v_F \propto n^{1/2}$
(D) $v_F \propto n^{1/3}$

Q.No:2 GATE-2012

The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the CORRECT equation of state (P being the pressure),

(A) $PV = \frac{1}{3}E$
(B) $PV = \frac{2}{3}E$
(C) $PV = E$
(D) $PV = \frac{5}{3}E$

Q.No:3 GATE-2012

Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids?

(A) Debye's T^3 law
(B) Dulong Petit's law
(C) Optical branches in lattices
(D) Thermal expansion

Q.No:4 GATE-2013

If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be (k is the Boltzmann constant and T is nonzero)

(A) Zero
(B) Infinite
(C) $\frac{3}{2}kT$
(D) kT

Q.No:5 GATE-2013

The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is _____.

Q.No:6 GATE-2014

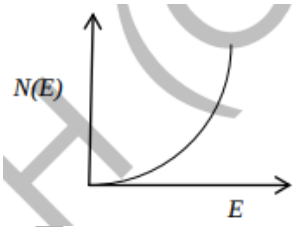
Which one of the following is a fermion?

(A) α particle
(B) ${}_4Be^7$ nucleus
(C) Hydrogen atom
(D) Deuteron

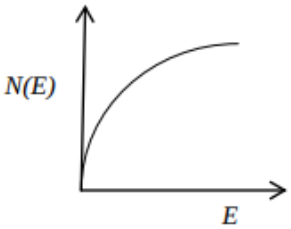
Q.No:7 GATE-2014

For a free electron gas in two dimensions, the variation of the density of states, $N(E)$ as a function of energy E, is best represented by

(A)



(B)



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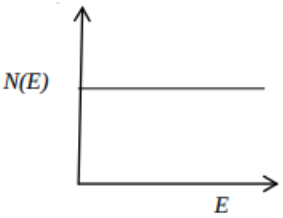

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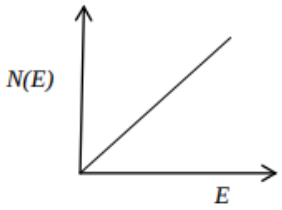

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Q.No:8 GATE-2014

Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(A) $k_B \ln 2$
(B) $2k_B \ln 2$
(C) $2k_B \ln 4$
(D) $3k_B \ln 4$

Q.No:9 GATE-2015

In Bose-Einstein condensates, the particles

(A) have strong interparticle attraction
(B) condense in real space
(C) have overlapping wavefunctions
(D) have large and positive chemical potential

Q.No:10 GATE-2015

For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because

(A) the chemical potential of the photons is zero
(B) photons obey Pauli exclusion principle
(C) photons are spin-1 particles
(D) the entropy of the photons is very large

Q.No:11 GATE-2015

The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is _____.

Q.No:12 GATE-2015

Given that the Fermi energy of gold is 5.54 eV, the number density of electrons is _____ $\times 10^{28} m^{-3}$ (upto one decimal place)

(Mass of electron $= 9.11 \times 10^{-31} kg$; $h = 6.626 \times 10^{-34} J.s$; $1eV = 1.6 \times 10^{-19} J$)

Q.No:13 GATE-2016

Consider a metal which obeys the Sommerfeld model exactly. If E_F is the Fermi energy of the metal at $T = 0K$ and R_H is its Hall coefficient, which of the following statements is correct?

(A) $R_H \propto E_F^{3/2}$
(B) $R_H \propto E_F^{2/3}$
(C) $R_H \propto E_F^{-3/2}$
(D) R_H is independent of E_F

Q.No:14 GATE-2016

Consider a system having three energy levels with energies $0, 2\varepsilon$ and 3ε, with respective degeneracies of $2, 2$ and 3. Four bosons of spin zero have to be accommodated in these levels such that the total energy of the system is 10ε. The number of ways in which it can be done is _____.

Q.No:15 GATE-2016

The Fermi energies of two metals X and Y are $5eV$ and $7eV$ and their Debye temperatures are $170K$ and $340K$, respectively. The molar specific heats of these metals at constant volume at low temperatures can be written as $(C_V)_X = \gamma_X T + A_X T^3$ and $(C_V)_Y = \gamma_Y T + A_Y T^3$, where γ and A are constants. Assuming that the thermal effective mass of the electrons in the two metals are same, which of the following is correct?

(A) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = 8$
(B) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = \frac{1}{8}$
(C) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = \frac{1}{8}$
(D) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = 8$

Q.No:16 GATE-2017

Consider a 2-dimensional electron gas with a density of $10^{19} m^{-2}$. The Fermi energy of the system is _____ eV (up to two decimal places).

Q.No:17 GATE-2017

The energy density and pressure of a photon gas are given by $u = aT^4$ and $P = u/3$, where T is the temperature and a is the radiation constant. The entropy per unit volume is given by $\alpha a T^3$. The value of α is _____. (up to two decimal places).

Q.No:18 GATE-2017

Consider two particles and two non-degenerate quantum levels 1 and 2. Level 1 always contains a particle. Hence, what is the probability that level 2 also contains a particle for each of the two cases:

(i) when the two particles are distinguishable and
(ii) when the two particles are bosons?

(A) (i) 1/2 and (ii) 1/3
(B) (i) 1/2 and (ii) 1/2
(C) (i) 2/3 and (ii) 1/2
(D) (i) 1 and (ii) 0

Q.No:19 GATE-2018

At low temperatures (T), the specific heat of common metals is described by (with α and β as constants)

(A) $\alpha T + \beta T^3$
(B) βT^3

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(C) $\exp(-\alpha/T)$
(D) $\alpha T + \beta T^5$

Q.No:20 GATE-2018

If X is the dimensionality of a free electron gas, the energy (E) dependence of density of states is given by $E^{\frac{1}{2}X-Y}$, where Y is _____.

Q.No:21 GATE-2018

Three particles are to be distributed in four non-degenerate energy levels. The possible number of ways of distribution:
(i) for distinguishable particles, and
(ii) for identical Bosons, respectively, is
(A) (i) 24, (ii) 4
(B) (i) 24, (ii) 20
(C) (i) 64, (ii) 20
(D) (i) 64, (ii) 16

Q.No:22 GATE-2019

A large number N of ideal bosons, each of mass m, are trapped in a three-dimensional potential $V(r) = \frac{m\omega^2 r^2}{2}$. The bosonic system is kept at temperature T which is much lower than the Bose-Einstein condensation temperature T_c. The chemical potential (μ) satisfies
(A) $\mu \leq \frac{3}{2}\hbar\omega$
(B) $2\hbar\omega > \mu > \frac{3}{2}\hbar\omega$
(C) $3\hbar\omega > \mu > 2\hbar\omega$
(D) $\mu = 3\hbar\omega$

Q.No:23 GATE-2019

The energy-wavevector (E-k) dispersion relation for a particle in two dimensions is $E = Ck$, where C is a constant. If its density of states $D(E)$ is proportional to E^p then the value of p is _____.

Q.No:24 GATE-2019

At temperature T Kelvin (K), the value of the Fermi function at an energy $0.5eV$ above the Fermi energy is 0.01. Then T, to the nearest integer, is _____ ($k_B = 8.62 \times 10^{-5} eV/K$)

Q.No:25 GATE-2020

Choose the correct statement related to the Fermi energy (E_F) and the chemical potential (μ) of a metal
(A) $\mu = E_F$ only at $0K$
(B) $\mu = E_F$ at finite temperature
(C) $\mu < E_F$ at $0K$
(D) $\mu < E_F$ at $0K$

Q.No:26 GATE-2021

For a finite system of Fermions where the density of states increases with energy, the chemical potential
(A) decreases with temperature
(B) increases with temperature
(C) does not vary with temperature
(D) corresponds to the energy where the occupation probability is 0.5

Q.No:27 GATE-2021

A system of two atoms can be in three quantum states having energies $0, \epsilon$ and 2ϵ. The system is in equilibrium at temperature $T = (k_B\beta)^{-1}$. Match the following **Statistics** with the **Partition function**.

Statistics	Partition function
CD: Classical (distinguishable particles)	**Z1**: $e^{-\beta\epsilon} + e^{-2\beta\epsilon} + e^{-3\beta\epsilon}$
CI: Classical (indistinguishable particles)	**Z2**: $1 + e^{-\beta\epsilon} + 2e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$
FD: Fermi-Dirac	**Z3**: $1 + 2e^{-\beta\epsilon} + 3e^{-2\beta\epsilon} + 2e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$
BE: Bose-Einstein	**Z4**: $\frac{1}{2} + e^{-\beta\epsilon} + \frac{3}{2}e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + \frac{1}{2}e^{-4\beta\epsilon}$

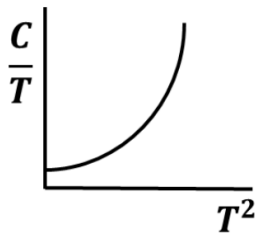
(A) **CD : Z1, CI : Z2, FD : Z3, BE : Z4**
(B) **CD : Z2, CI : Z3, FD : Z4, BE : Z1**
(C) **CD : Z3, CI : Z4, FD : Z1, BE : Z2**
(D) **CD : Z4, CI : Z1, FD : Z2, BE : Z3**

Q.No:28 GATE-2022

In a two-dimensional square lattice, frequency ω of phonons in the long wavelength limit changes linearly with the wave vector k. Then the density of states of phonons is proportional to
(A) ω
(B) ω^2
(C) $\sqrt{\omega}$
(D) $\frac{1}{\sqrt{\omega}}$

Q.No:29 GATE-2023

For a non-magnetic metal, which one of the following graphs best represents the behaviour of $\frac{C}{T}$ vs. T^2, where C is the heat capacity and T is the temperature?
(A)



(B)

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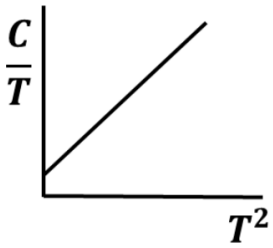

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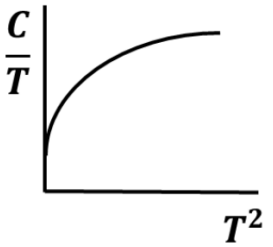

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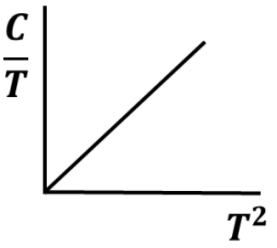


Q.No:30 GATE-2023

Two identical, non-interacting $^4He_2$ atoms are distributed among 4 different nondegenerate energy levels. The probability that they occupy different energy levels is p. Similarly, two $^3He_2$ atoms are distributed among 4 different non-degenerate energy levels, and the probability that they occupy different levels is q. What is the value of $\frac{p}{q}$ (rounded off to one decimal place)?

Q.No:31 GATE-2024

Consider a three-dimensional system of non-interacting bosons with zero chemical potential. The energy of the system $\varepsilon \propto k^2$, where k is the wavevector. The low temperature specific heat of the system at constant volume depends on the temperature as $C_v \propto T^{\frac{n}{2}}$. The value of n is _____ (in integer).

Q.No:32 GATE-2024

The canonical partition function of an ideal gas is

$$Q(T,V,N) = \frac{1}{N!}\left[\frac{V}{(\lambda(T))^3}\right]^N$$

where T, V, N, and $\lambda(T)$ denote temperature, volume, number of particles, and thermal de Broglie wavelength, respectively. Let k_B be the Boltzmann constant and μ be the chemical potential. Take $\ln(N!) = N\ln(N) - N$.

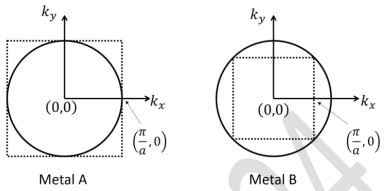
If the number density $\left(\frac{N}{V}\right)$ is $2.5 \times 10^{25}\,\mathrm{m}^{-3}$ at a temperature T, then the value of

$$\frac{e^{\mu/(k_B T)}}{(\lambda(T))^3} \times 10^{-25}$$

is _____ m^{-3} (rounded off to one decimal place).

Q.No:33 GATE-2024

Crystal structures of two metals A and B are two-dimensional square lattices with the same lattice constant a. Electrons in metals behave as free electrons. The Fermi surfaces corresponding to A and B are shown by solid circles in figures.



The electron concentrations in A and B are n_A and n_B, respectively. The value of $\frac{n_B}{n_A}$ is

(A) 3
(B) 2
(C) $3\sqrt{3}$
(D) $\sqrt{2}$

Q.No:34 GATE-2025

Consider one mole of a monovalent metal at absolute zero temperature, obeying the free electron model. Its Fermi energy is E_F. The energy corresponding to the filling of $\frac{N_A}{2}$ electrons, where N_A is the Avogadro number, is $2^n E_F$. The value of n is

(A) $-\frac{2}{3}$
(B) $+\frac{2}{3}$
(C) $-\frac{1}{3}$
(D) -1

Q.No:35 GATE-2025

Consider a monatomic chain of length 30 cm. The phonon density of states is 1.2×10^{-4} s. Assuming the Debye model, the velocity of sound in m/s (rounded off to one decimal place) is _____.

Q.No:36 GATE-2025

A system of five identical, non-interacting particles with mass m and spin $\frac{3}{2}$ is confined to a one-dimensional potential well of length L. If the lowest energy of the system

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is

$$N\frac{\pi^2\hbar^2}{2mL^2},$$

the value of N (in integer) is _____.

Q.No:37 GATE-2026

The formula for energy E of a photon gas at temperature T in a two-dimensional box at equilibrium with $g_{2d}(\nu)$ denoting the density of states of photons is given below where symbols ν, h and k_B have their standard meaning.

$$E = \int_0^\infty d\nu \; g_{2d}(\nu)\frac{h\nu}{\exp\left(\frac{h\nu}{k_BT}\right)-1}$$

The specific heat (C_V) of this photon gas obeys
(A) $C_V \propto T$
(B) $C_V \propto T^2$
(C) $C_V \propto T^3$
(D) $C_V \propto T^4$

Q.No:38 GATE-2026

A gas of non-interacting ^{4}He atoms (of mass m) is in a three-dimensional trap whose energy levels can be approximated by those of a harmonic oscillator potential

$$V(x,y,z) = \frac{1}{2}m\omega^2(x^2+y^2+z^2).$$

The chemical potential of the gas at $T = 0$ K is
(A) 0
(B) $\frac{1}{2}\hbar\omega$
(C) $\frac{3}{2}\hbar\omega$
(D) $3\hbar\omega$

Q.No:39 GATE-2026

The specific heat $C_p(T)$ of one mole of a material as a function of temperature T is given as

$$C_p(T) = AT + BT^3,$$

where

$$A = 0.695 \text{ mJ.mol}^{-1}\text{.K}^{-2}$$

and

$$B = 0.045 \text{ mJ.mol}^{-1}\text{.K}^{-4}.$$

When T is changed from 1 K to 10 K at constant pressure, then the change in entropy ΔS in mJ.mol^{-1}.K^{-1} (rounded off to one decimal place) is _____.

Q.No:40 GATE-2026

Two identical particles with a fixed total energy

$$E = 2\hbar\omega$$

are in thermal equilibrium in a one-dimensional harmonic oscillator potential

$$\frac{1}{2}m\omega^2x^2.$$

Let the entropy of the particles be denoted by S_F if they are fermions with spin $\frac{1}{2}$ ($S_z = \pm\frac{\hbar}{2}$) and S_B if they are bosons with spin 0. Then, which of the following options is correct?
(k_B is the Boltzmann constant)
(A) $S_F = k_B \ln 2, \quad S_B = k_B \ln 2$
(B) $S_F = 2k_B \ln 2, \quad S_B = 0$
(C) $S_F = 4k_B \ln 2, \quad S_B = 0$
(D) $S_F = 2k_B \ln 2, \quad S_B = k_B \ln 2$

Q.No:41 GATE-2026

Copper has an electron number density of 8.3×10^{28} m^{-3}. Its Fermi energy in eV (rounded off to one decimal place) is _____. ($\hbar = 1.06 \times 10^{-34}$J.s, $m_e = 9.10 \times 10^{-31}$ kg, charge of electron $= 1.60 \times 10^{-19}$ C)

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Question No.	**Entrance with year**	answer
Q1	GATE-2012	Option D
Q2	GATE-2012	Option B
Q3	GATE-2012	Option D
Q4	GATE-2013	Option B
Q5	GATE-2013	70
Q6	GATE-2014	Option B
Q7	GATE-2014	Option C
Q8	GATE-2014	Option B
Q9	GATE-2015	Option C
Q10	GATE-2015	Option A
Q11	GATE-2015	0
Q12	GATE-2015	5.9–6.0
Q13	GATE-2016	Option C
Q14	GATE-2016	18
Q15	GATE-2016	Option A
Q16	GATE-2017	2.32–2.40
Q17	GATE-2017	1.30–1.36
Q18	GATE-2017	Option C
Q19	GATE-2018	Option A
Q20	GATE-2018	1
Q21	GATE-2018	Option C
Q22	GATE-2019	Option A
Q23	GATE-2019	1
Q24	GATE-2019	1260–1266
Q25	GATE-2020	Option A
Q26	GATE-2021	Option A & D
Q27	GATE-2021	Option C
Q28	GATE-2022	Option A
Q29	GATE-2023	Option B
Q30	GATE-2023	0.6
Q31	GATE-2024	3
Q32	GATE-2024	2.5
Q33	GATE-2024	Option B
Q34	GATE-2025	Option A
Q35	GATE-2025	794–797
Q36	GATE-2025	8
Q37	GATE-2026	Option B
Q38	GATE-2026	Option C
Q39	GATE-2026	21.1–21.3
Q40	GATE-2026	Option B
Q41	GATE-2026	6.5–7.5

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