Lagrangian Mechanics JEST & TIFR

https://snboseplc.com/lagrangian-mechanics-jest-tifr/

1. JEST-2014

A double pendulum consists of two equal masses m suspended by two strings of length l. What is the Lagrangian of this system for oscillations in a plane? Assume the angles θ_1, θ_2 made by the two strings are small (you can use $\cos\theta = 1 - \theta^2/2$).
Note : $\omega_0 = \sqrt{g/l}$.

(A) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(B) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(C) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(D) $L \approx ml^2\left(\frac{1}{2}\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \omega_0^2\theta_2^2\right)$

2. JEST-2014

Let us write down the Lagrangian of a system as $L(x, \dot{x}, \ddot{x}) = mx\ddot{x} + kx^2 + cx\dddot{x}$. What is the dimension of c?

(A) MLT^{-3}

(B) MT^{-2}

(C) MT

(D) ML^2T^{-1}

3. JEST-2015

A bike stuntman rides inside a well of frictionless surface given by $z = a(x^2+y^2)$, under the action of gravity acting in the negative z direction. $\vec{g} = -g\hat{z}$ What speed should he maintain to be able to ride at a constant height z_0 without falling down?

(A) $\sqrt{gz_0}$

(B) $\sqrt{3gz_0}$

(C) $\sqrt{2gz_0}$

(D) The biker will not be able to maintain a constant height, irrespective of speed.

4. JEST-2015

The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. Which of the following statements is true?

(A) This is a free particle

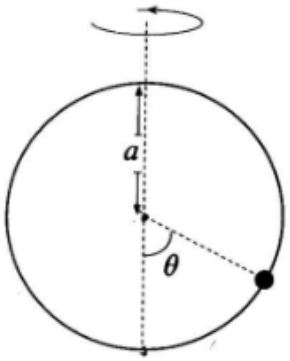
(B) The particle is experiencing velocity dependent damping

(C) The particle is executing simple harmonic motion

(D) The particle is under constant acceleration.

5. JEST-2016

A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$, at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?



(A) $\tan\theta = \pi g/\omega^2 a$

(B) $\sin\theta = g/\omega^2 a$

(C) $\cos\theta = g/\omega^2 a$

(D) $\tan\theta = g/\pi\omega^2 a$

6. JEST-2017

A possible Lagrangian for a free particle is

(A) $L = \dot{q}^2 - q^2$.

(B) $L = \dot{q}^2 - q\dot{q}$.

(C) $L = \dot{q}^2 - q$.

(D) $L = \dot{q}^2 - \frac{1}{q}$.

7. JEST-2017

A bead of mass M slides along a parabolic wire described by $z = 2(x^2+y^2)$. The wire rotates with angular velocity Ω about the z-axis. At what value of Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$?

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(A) $\sqrt{3g}$

(B) $\sqrt{g}$

(C) $\sqrt{2g}$

(D) $\sqrt{4g}$

8. JEST-2017

A rod of mass m and length l is suspended from two massless vertical springs with a spring constants k_1 and k_2. What is the Lagrangian for the system, if x_1 and x_2 be the displacements from equilibrium position of the two ends of the rod?

(A) $\frac{m}{8}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

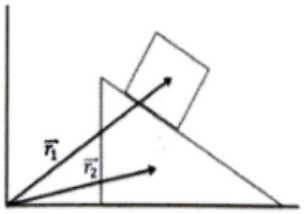
(B) $\frac{m}{2}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 + k_2)(x_1^2 + x_2^2)$

(C) $\frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(D) $\frac{m}{4}(\dot{x}_1^2 - 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 - k_2)(x_1^2 + x_2^2)$

9. JEST-2018

A block of mass M is moving on a frictionless inclined surface of a wedge of mass m under the influence of gravity. The wedge is lying on a rigid frictionless horizontal surface. The configuration can be described using the radius vectors $\vec{r}_1$ and $\vec{r}_2$ shown in the figure. How many constraints are present and what are the types?



(A) One constraint; holonomic and scleronomous

(B) Two constraints; Both are holonomic; one is scleronomous and rheonomous

(C) Two constraints; Both are scleronomous; one is holonomic and the other is non-holonomic.

(D) Two constraints; Both are holonomic and scleronomous

10. JEST-2018

Consider the Lagrangian

$$L = 1 - \sqrt{1 - \dot{q}^2} - \frac{q^2}{2}$$

of a particle executing oscillations whose amplitude is A. If p denotes the momentum of the particle, then $4p^2$ is

(A) $(A^2 - q^2)(4 + A^2 - q^2)$

(B) $(A^2 + q^2)(4 + A^2 - q^2)$

(C) $(A^2 - q^2)(4 + A^2 + q^2)$

(D) $(A^2 + q^2)(4 + A^2 + q^2)$

11. JEST-2019

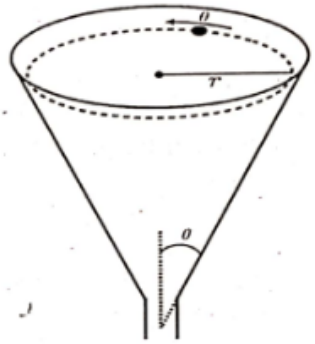
Consider the motion of a particle in two dimensions given by the Lagrangian

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{\lambda}{4}(x + y)^2,$$

where $\lambda > 0$. The initial conditions are given as $y(0) = 0, x(0) = 42$ meters, $\dot{x}(0) = \dot{y}(0) = 0$. What is the value of $x(t) - y(t)$ at $t = 25$ seconds in meters?

12. JEST-2020

A particle is to slide along the horizontal circular path on the inner surface of the funnel as shown in the figure. The surface of the funnel is frictionless. What must be the speed of the particle (in terms of r and θ) if it is to execute this motion?



(A) $\sqrt{rg\sin\theta}$

(B) $\sqrt{rg\cos\theta}$

(C) $\sqrt{rg\tan\theta}$

(D) $\sqrt{rg\cot\theta}$

13. JEST-2021

A particle of mass m is subject to the potential $V(x, y, t) = K(x^2 + y^2)$, where (x, y) are the Cartesian coordinates of the particle and K is a constant. Which one of the following quantities is a constant of motion?

(A) $\dot{y}x + \dot{x}y$

(B) $\dot{y}x - \dot{x}y$

(C) $\dot{y} + \dot{x}$

(D) $\dot{y}y + \dot{x}x$

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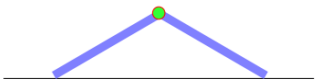
14. JEST-2022

Two uniform rods of length $1m$ are connected to a friction-less hinge A. The hinge is held at a height and the other ends of the rods rests on a friction-less plane, such that the angle between the rods is $2\pi/3$. If the hinge is released from the rest, what is the speed of the hinge when it hits the floor? [Acceleration due to gravity is $9.81ms^{-2}$]



15. JEST-2023

The action corresponding to the motion of a particle in one dimension is:

$$S = \int_{t_i}^{t_f} dt \left[\frac{1}{2}m\dot{x}^2 - V(x) + \alpha x\ddot{x} + \beta x\dot{x}\right]$$

where m is the mass of the particle, α, β are constants, and $V(x)$ is a potential which is a function of x. The position and velocity are held fixed at the end points of the trajectory. The equation of motion of the particle is

(A) $(2\alpha + m)\ddot{x} - \frac{dV}{dx} = 0$

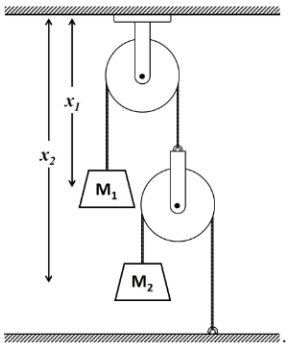
(B) $(2\alpha - m)\ddot{x} + \beta\dot{x} - \frac{dV}{dx} = 0$

(C) $(2\alpha - m)\ddot{x} - \beta\dot{x} - \frac{dV}{dx} = 0$

(D) $(2\alpha - m)\ddot{x} - \frac{dV}{dx} = 0$

16. JEST-2024

Consider a mass-pulley system as shown in the figure. The heights of the blocks as measured from the ceiling are x_1 and x_2, as shown in the figure.
What is the constraint between x_1 and x_2?



(A) $x_2 + 2x_1 =$ constant.

(B) $x_2 - x_1 =$ constant.

(C) $x_2 + x_1 =$ constant.

(D) They are unconstrained.

17. JEST-2024

Two classical particles moving in three dimensions interact via the potential

$$V = K\left[(x_1^2 + y_1^2) + (x_2^2 + y_2^2) + (z_1 - z_2)^2\right],$$

where K is a constant, and (x_1, y_1, z_1) and (x_2, y_2, z_2) are the Cartesian coordinates of the two particles. Let (p_1^x, p_1^y, p_1^z) and (p_2^x, p_2^y, p_2^z) be the components of the linear momenta of the two particles, and (L_1^x, L_1^y, L_1^z) and (L_2^x, L_2^y, L_2^z) the components of the corresponding angular momenta. Which of the following statements is true?

(A) L_1^z, L_2^z, and $(p_1^z + p_2^z)$ are conserved.

(B) L_1^z and L_2^z are not separately conserved but $L_1^z + L_2^z$ is conserved.

(C) $(L_1^x + L_2^x), (L_1^y + L_2^y), (L_1^z + L_2^z)$ are conserved.

(D) $(L_1^x + L_2^x)$ and $(L_1^y + L_2^y)$ are conserved.

18. JEST-2024

A classical system has the following action:

$$S = \int (\dot{q}^2 + \alpha q\dot{q} + \beta q^2\dot{q})dt,$$

where q is the generalized coordinate, and α and β are constants. Which of the following statements is true about the dynamics of the system?

(A) The dynamics is independent of α and β.

(B) The dynamics depends only on α.

(C) The dynamics depends only on β.

(D) The dynamics depends on ratio α/β.

19. JEST-2025

The Lagrangian of a two-dimensional system is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - k(x^2 + y^2)^{-1.5}.$$

Which of the following is/are the constant(s) of motion?

(A) Angular momentum only

(B) Energy only

(C) Energy and angular momentum

(D) None of the others

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20. JEST-2026

Consider the action

$$S = \int_0^2 \left[\left(\frac{dx}{dt} \right)^2 + 6x \right] dt.$$

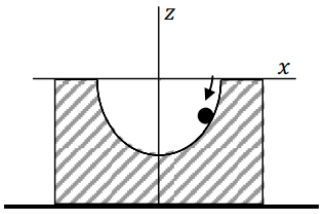
If the boundary conditions are such that $x(0) = 0$, and $x(2) = 6$, what is the value of the action evaluated on the extremal path?

21. JEST-2026

The Lagrangian of a particle in one dimension is $L = 10\dot{x}^2 + 6\dot{x}t^2 + \alpha xt$ where $\dot{x} \equiv \frac{dx}{dt}$ and α is a constant. The particle starts moving at time $t = 0$ from the position $x(0) = 0$ with $\dot{x}(0) = 0.5$ in appropriate units. For what value of α is the position of the particle at time t given by $x(t) = \frac{t+1}{2}$?

1. TIFR-2013

A ball of mass m slides under gravity without friction inside a semicircular depression of radius a inside a fixed block placed on a horizontal surface, as shown in the figure. The equation of motion of the ball in the x-direction will be



(A) $\ddot{x} = \frac{g}{a} x \sqrt{1 - \frac{x^2}{a^2}}$

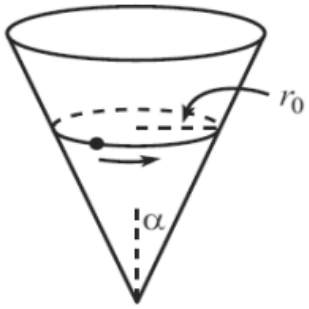
(B) $\ddot{x} = \frac{g}{a} x$

(C) $\ddot{x} = -\frac{g}{a} x$

(D) $\ddot{x} = -\frac{g}{a} x \sqrt{1 - \frac{x^2}{a^2}}$

2. TIFR-2015

A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is α. If the particle moves in a circle of radius r_0, without slipping downwards, the angular frequency ω of this motion will be



(A) $\sqrt{\frac{g}{r_0 \cos\alpha}}$

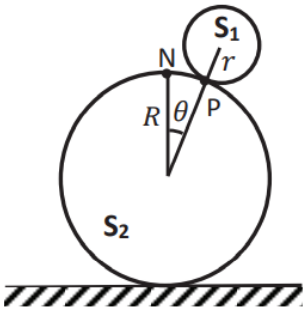
(B) $\sqrt{\frac{g}{r_0 \sin\alpha}}$

(C) $\sqrt{\frac{g}{r_0 \cot\alpha}}$

(D) $\sqrt{\frac{g}{r_0 \tan\alpha}}$

3. TIFR-2017

A uniform solid sphere S_1 of radius r and mass m is rolling without slipping on top of another sphere S_2 of radius R, as shown in the figure. Initially, S_1 was at rest directly on top of S_2, and then it started rolling down under the influence of gravity. The point of contact P subtends an instantaneous angle θ from the topmost point N of the lower sphere at the centre of the lower sphere.



At what minimum value of θ will the spheres lose contact?

(A) $\cos^{-1} \frac{5}{12}$

(B) $\cos^{-1} \frac{5}{13}$

(C) $\cos^{-1} \frac{2}{3}$

(D) $\cos^{-1} \frac{12}{13}$

4. TIFR-2018

A particle of mass m moving in one-dimension x is subjected to the Lagrangian

$$L = \frac{1}{2} m (\dot{x} - \lambda x)^2$$

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where λ is a real constant. If it starts at the origin at $t = 0$, its motion corresponds to the equation (a is a constant)

(A) $x = a \exp \lambda t$

(B) $x = a\{1 - \exp(-\lambda t)\}$

(C) $x = a \sin \lambda t$

(D) $x = a \sinh \lambda t$

5. TIFR-2018

The Hamiltonian of a particle of charge q and mass m in an electromagnetic field is given by

$$H = \frac{1}{2m}|\vec{p} - q\vec{A}(\vec{x}, t)|^2 + q\varphi(\vec{x}, t)$$

where $(\varphi, \vec{A})$ are the electromagnetic potentials. Clearly this Hamiltonian changes under a gauge transformation

$$\varphi \to \varphi - \frac{\partial \chi}{\partial t} \vec{A} \to \vec{A} + \vec{\nabla}\chi$$

where $\chi(\vec{x}, t)$ is a gauge function. Nevertheless the motion of the particle is not affected because

(A) the Lagrangian does not change under the gauge transformation.

(B) the motion of the particle is correctly described only in the Lorenz gauge.

(C) the action of the particle changes only by surface terms which do not vary.

(D) the Lorentz force is modified to balance the effect of the gauge transformation.

6. TIFR-2021

A particle of mass m moves in a plane (r, θ) under the influence of a force

$$\vec{F} = \frac{mk}{r^3}(x\hat{r} + y\hat{\theta})$$

where $x = r\cos\theta$ and $y = r\sin\theta$, while k is a constant. The Lagrangian for this system is

(A) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 - \frac{kx}{r^2}\right)$

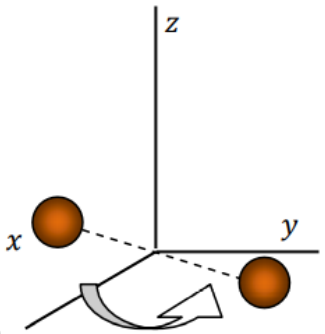
(B) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 - \frac{ky}{r^2}\right)$

(C) $L = \frac{1}{2}m\left(\frac{\dot{r}^2}{r} + r\dot{\theta}^2 - \frac{kxy}{r}\right)$

(D) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \frac{kx}{r^2}\right)$

7. TIFR-2021

Consider a diatomic molecule of oxygen which is rotating in the xy-plane about the z axis. The z axis passes through the centre of the molecule and is perpendicular to its length. At room temperature, the average separation between the two oxygen atoms is 1.21×10^{-10} m (the atoms are treated as point masses). The molar mass of oxygen is 16 gm/mol.



If the angular velocity of the molecule about the z axis is 2×10^{12} rad/s, its rotational kinetic energy will be closet to

(A) 3.89×10^{-22} Joule

(B) 7.78×10^{-22} Joule

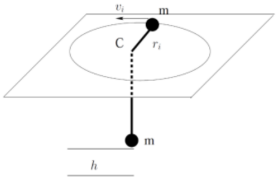
(C) 15.56×10^{-22} Joule

(D) 1.95×10^{-22} Joule

8. TIFR-2021

Two equal masses m are connected with a massless string. The first mass is initially set in a uniform circular motion with speed v_i, at a radius r_i on top of a table, while the second mass hangs vertically on the string which passes through a hole in the center of the table (C), as shown in the figure below.

The system is released with this initial configuration and the second mass starts falling. What is the net speed of the first mass when the second mass has fallen a height h (smaller than r_i)? (Assume that there is no friction and that the string always remains tight.)



(A) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \left(\frac{r_i^2}{(r_i - h)^2}\right)\right]}$

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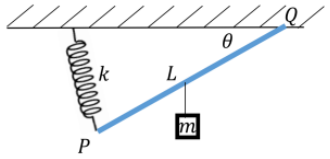
(B) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \left(\frac{r_i}{r_i - h}\right)\right]}$

(C) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \left(\frac{(r_i-h)^2}{r_i^2}\right)\right]}$

(D) $\sqrt{gh + \frac{1}{2}v_i^2\left[1 + \left(\frac{r_i - h}{r_i}\right)\right]}$

9. TIFR-2025

A massless rigid rod of length L is suspended with an ideal spring of springconstant k at one end P, and by a hinge on the other end Q.The rest length of the spring is zero. A mass m is suspended from themid-point of the rod. This results in tilting of the rod by angle θ.What is the angle θ?



(A) $\tan^{-1}\left(\frac{mg}{2kL}\right)$

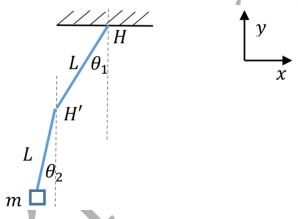
(B) $\sin^{-1}\left(\frac{mg}{2kL}\right)$

(C) $\cos^{-1}\left(\frac{mg}{kL}\right)$

(D) $\sec^{-1}\left(\frac{mg}{kL}\right)$

10. TIFR-2025

A composite pendulum consists of two massless rods and a weight m.The two rods are connected by a hinge H'.The other end of the first rod is connected to the ceiling by a hinge H.The rods can move freely about H, H' in the xy-plane.What is the Lagrangian of the system?



(A) $\frac{L^2m}{2}\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)\right) + gLm(\cos\theta_1 + \cos\theta_2)$

(B) $\frac{L^2m}{2}\left(\dot{\theta}_1^2 + \dot{\theta}_2^2\right) + gLm(\cos\theta_1 + \cos\theta_2)$

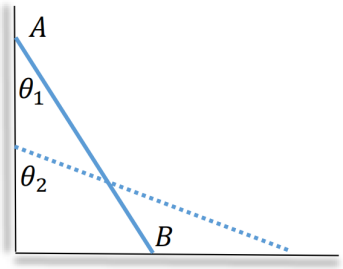
(C) $\frac{L^2m}{2}\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2)\right) - gLm(\cos\theta_1 + \cos\theta_2)$

(D) $\frac{L^2m}{2}\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 - 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)\right) - gLm(\cos\theta_1 + \cos\theta_2)$

11. TIFR-2026

Consider a uniform rod with length L and mass m with one end A on the wall and the other end B on the ground. Initially it is at rest at an angle θ_1 with the wall and starts slipping. What is the speed of A when the ladder's angle with the wall is θ_2? Ignore friction.

(FYI: The moment of inertia of a uniform rod about its centre is $I = \frac{1}{12}mL^2$.)



(A) $\sin\theta_2\sqrt{3gL(\cos\theta_1 - \cos\theta_2)}$

(B) $\sqrt{3gL(\cos\theta_1 - \cos\theta_2)}$

(C) $\sqrt{12gL(\cos\theta_1 - \cos\theta_2)}$

(D) $\sin\theta_2\sqrt{6gL(\cos\theta_1 - \cos\theta_2)}$

12. TIFR-2026

A classical particle of mass m moving in the $x - y$ plane is constrained to move along the curve

$$x^4 + y^4 = 1.$$

There are no other external forces acting on it. If (r, θ) are the polar co-ordinates, which of the following is true?

(A) $\ddot{\theta} + \frac{2\dot{r}\dot{\theta}}{r} \propto r^2\sin 4\theta$

(B) $\ddot{\theta} \propto r^2\sin 4\theta$

(C) $\ddot{\theta} \propto r^4\sin^2 2\theta$

(D) $\ddot{\theta} + \frac{2\dot{r}\dot{\theta}}{r} \propto r^4\sin^2 2\theta$

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Q No.	**Entrance with year**	answer
Q1	JEST-2014	Option B
Q2	JEST-2014	Option C
Q3	JEST-2015	Option C
Q4	JEST-2015	Option A
Q5	JEST-2016	Option C
Q6	JEST-2017	Option B
Q7	JEST-2017	Option D
Q8	JEST-2017	Option C
Q9	JEST-2018	Option D
Q10	JEST-2018	Option A
Q11	JEST-2019	42
Q12	JEST-2020	Option D
Q13	JEST-2021	Option B
Q14	JEST-2022	1.92
Q15	JEST-2023	Option D
Q16	JEST-2024	Option A
Q17	JEST-2024	Option A
Q18	JEST-2024	Option A
Q19	JEST-2025	Option C
Q20	JEST-2026	48
Q21	JEST-2026	12

Q No.	**Entrance with year**	answer
Q1	TIFR-2013	Option D
Q2	TIFR-2015	Option D
Q3	TIFR-2017	Option A
Q4	TIFR-2018	Option D
Q5	TIFR-2018	Option C
Q6	TIFR-2021	Option A
Q7	TIFR-2021	Option A
Q8	TIFR-2021	Option A
Q9	TIFR-2025	Option A
Q10	TIFR-2025	Option A
Q11	TIFR-2026	Option A
Q12	TIFR-2026	Option A

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